



*On the Job Training on  
Tsunami Inundation Modelling and Mapping and Development of Tsunami Hazards Maps for  
Implementation of UNESCO-IOC Tsunami Ready Pilot Sites in Madagascar, Maldives, Seychelles and Sri Lanka  
Hyderabad – India, 16–21 March 2026*

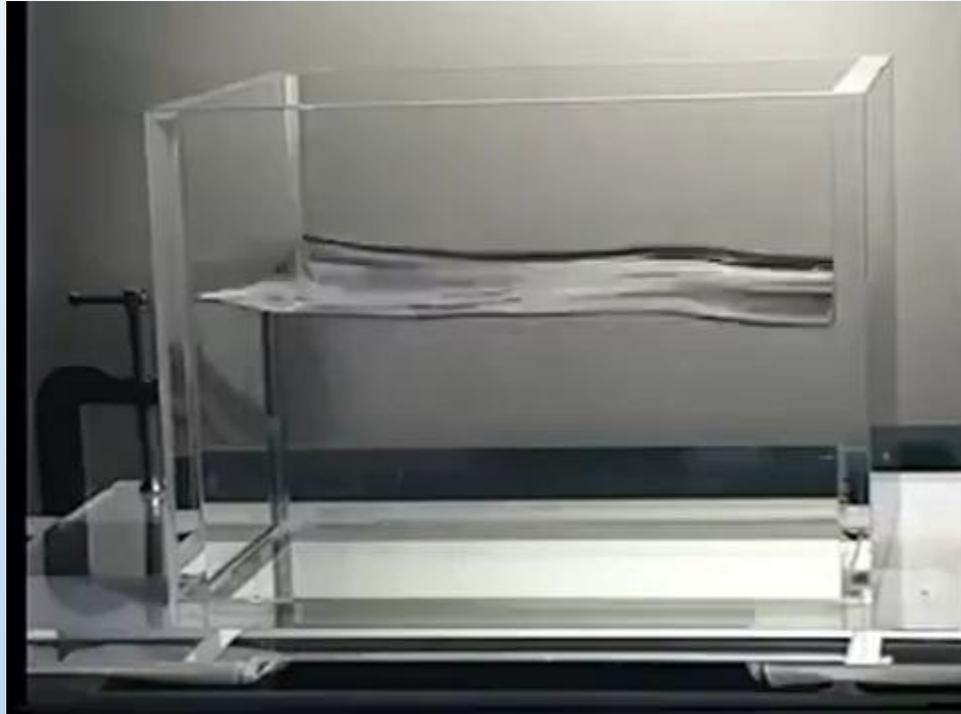
# **Tsunami Inundation Modelling and Mapping**

## ***TIMM: Tsunami Science, Numerical Modelling and Forecasting***

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# Sloshing



**Violent or noisy movement of liquid inside a partially filled container, such as tanks in ships, trucks, or rockets.**

# Instability on the free surface of fluid under external excitations (Tsunami Basic)

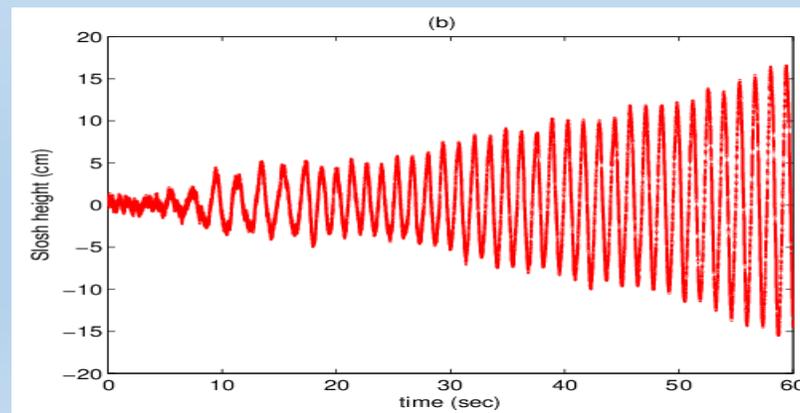
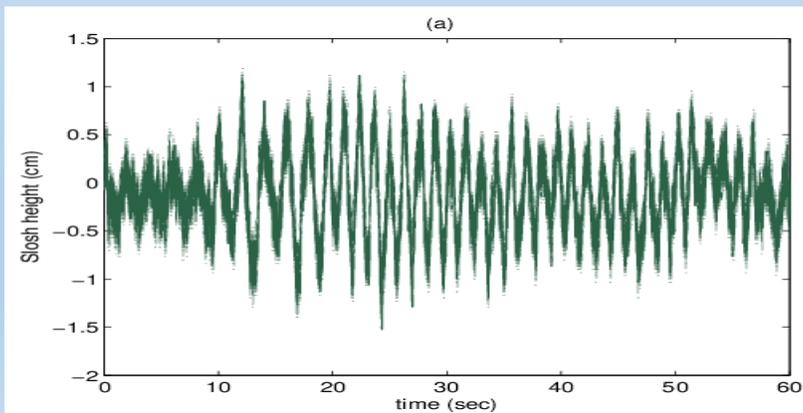
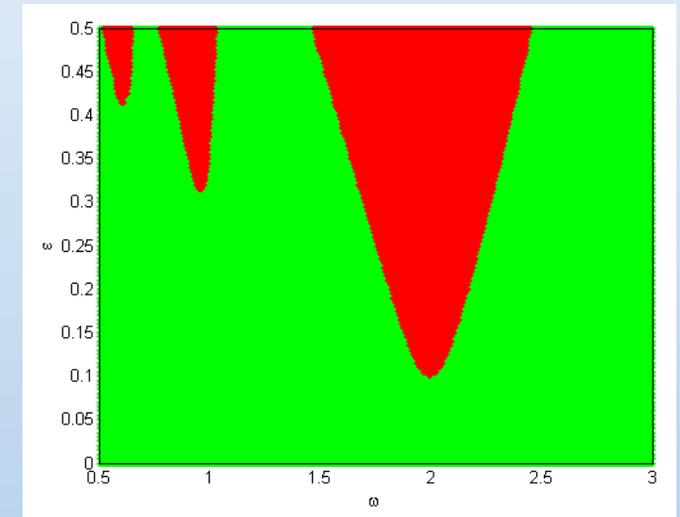
## Stability on free-surface



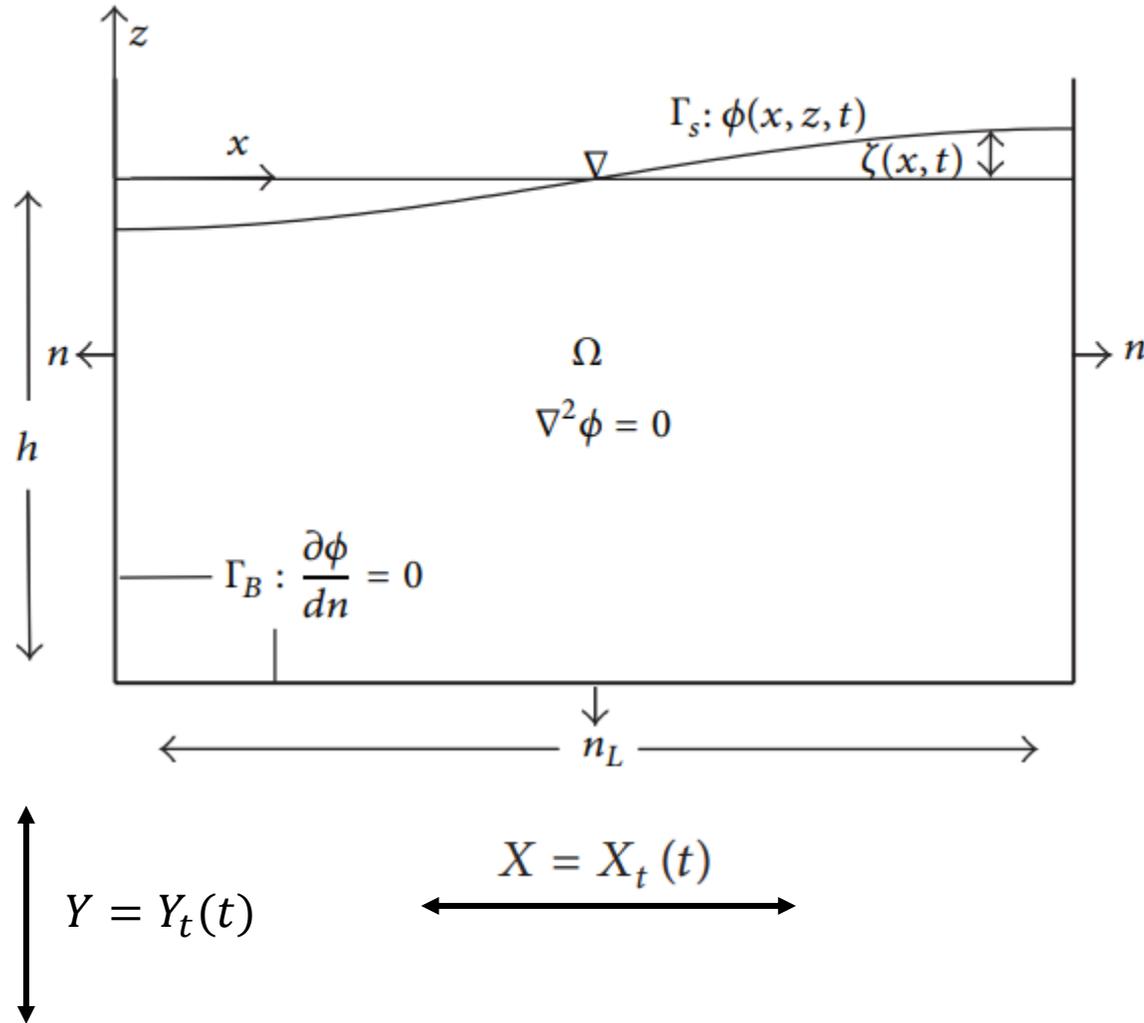
## Instability on free-surface



## Instability Chart



# Sloshing Governing Equations



$$\nabla^2 \phi = 0.$$

$$\left. \frac{\partial \phi}{\partial n} \right|_{x=0,L} = 0, \quad \left. \frac{\partial \phi}{\partial n} \right|_{z=-h} = 0.$$

$$\left. \frac{\partial \phi}{\partial t} \right|_{z=\zeta} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + g\zeta + x X_t'',$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} - \frac{\partial \phi}{\partial z} = 0.$$

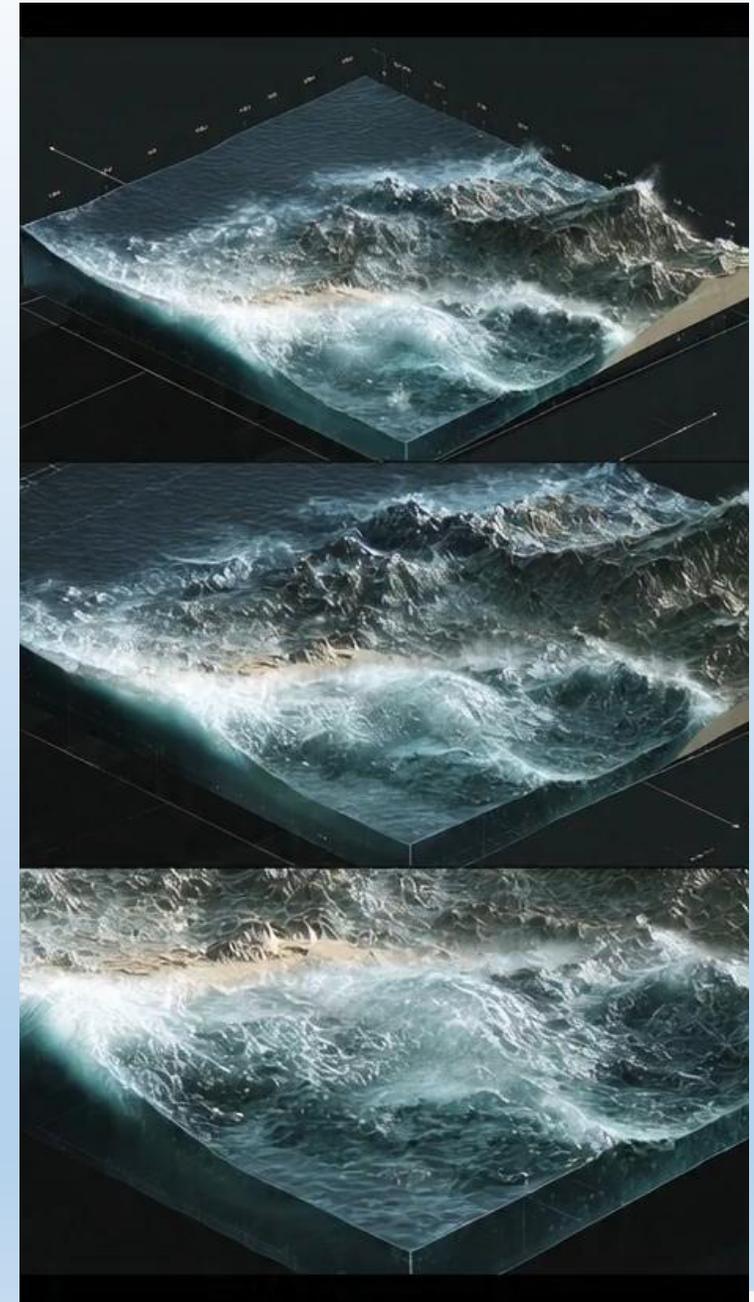
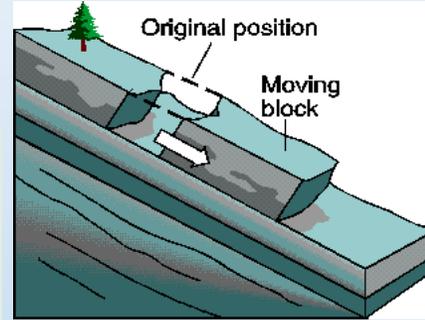
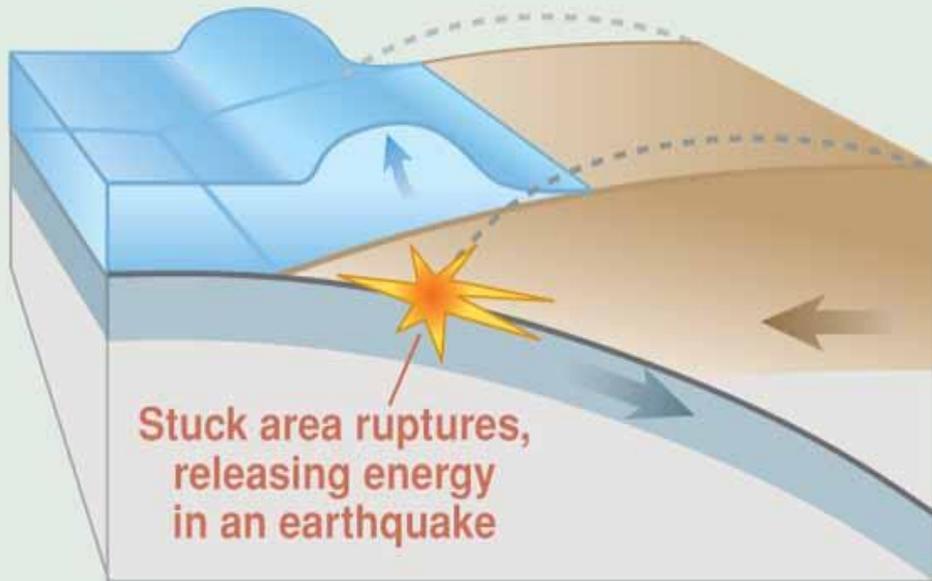
$$\phi(x, 0, 0) = -x \frac{dX_t(t)}{dt},$$

$$\zeta(x, 0) = 0.$$

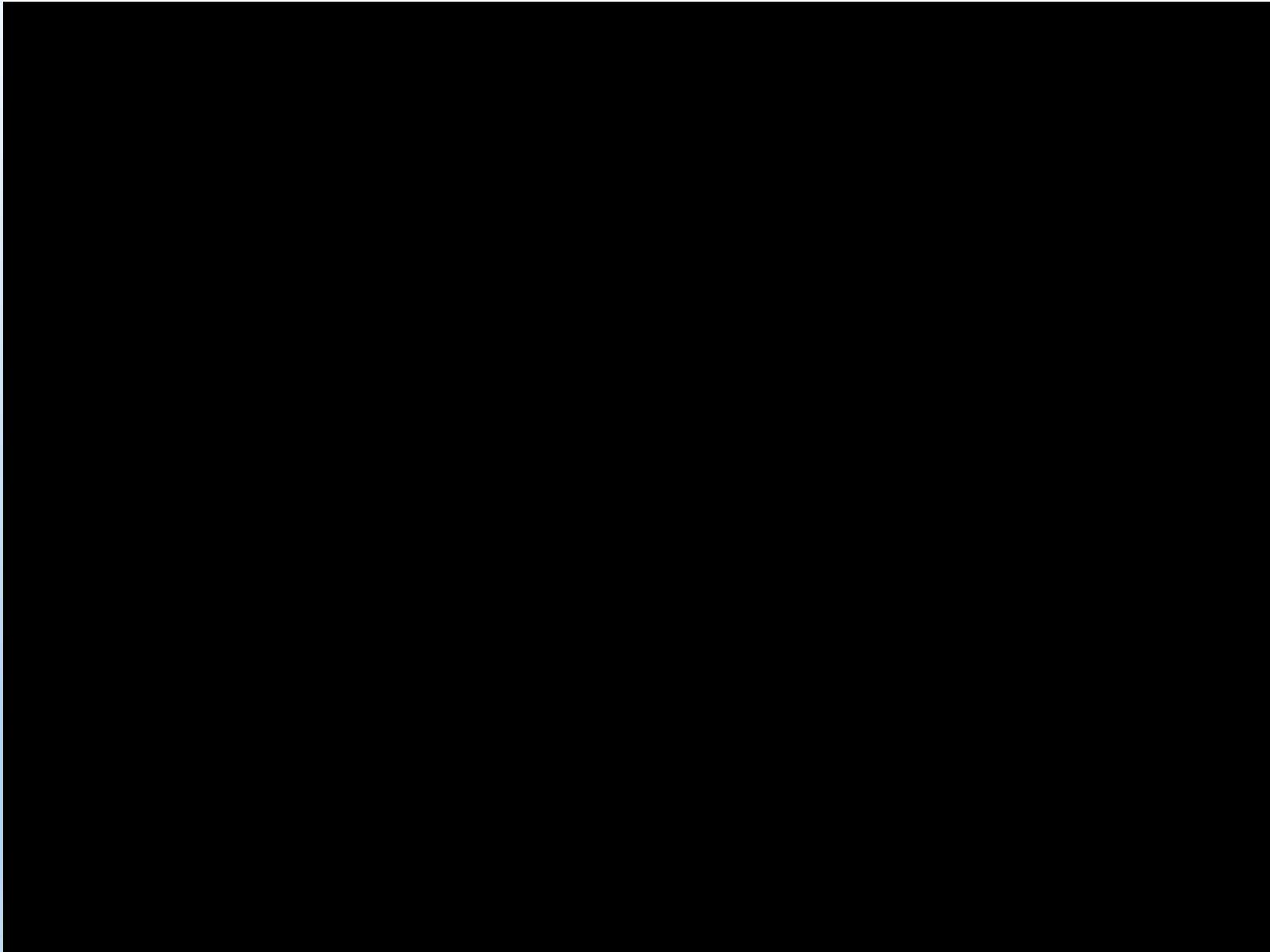
Any impulse may cause large-scale displacement of the sea surface.

- Earthquakes
- Landslide
- Volcanic eruptions
- Meteoroids Impact

*Earthquake starts tsunami*



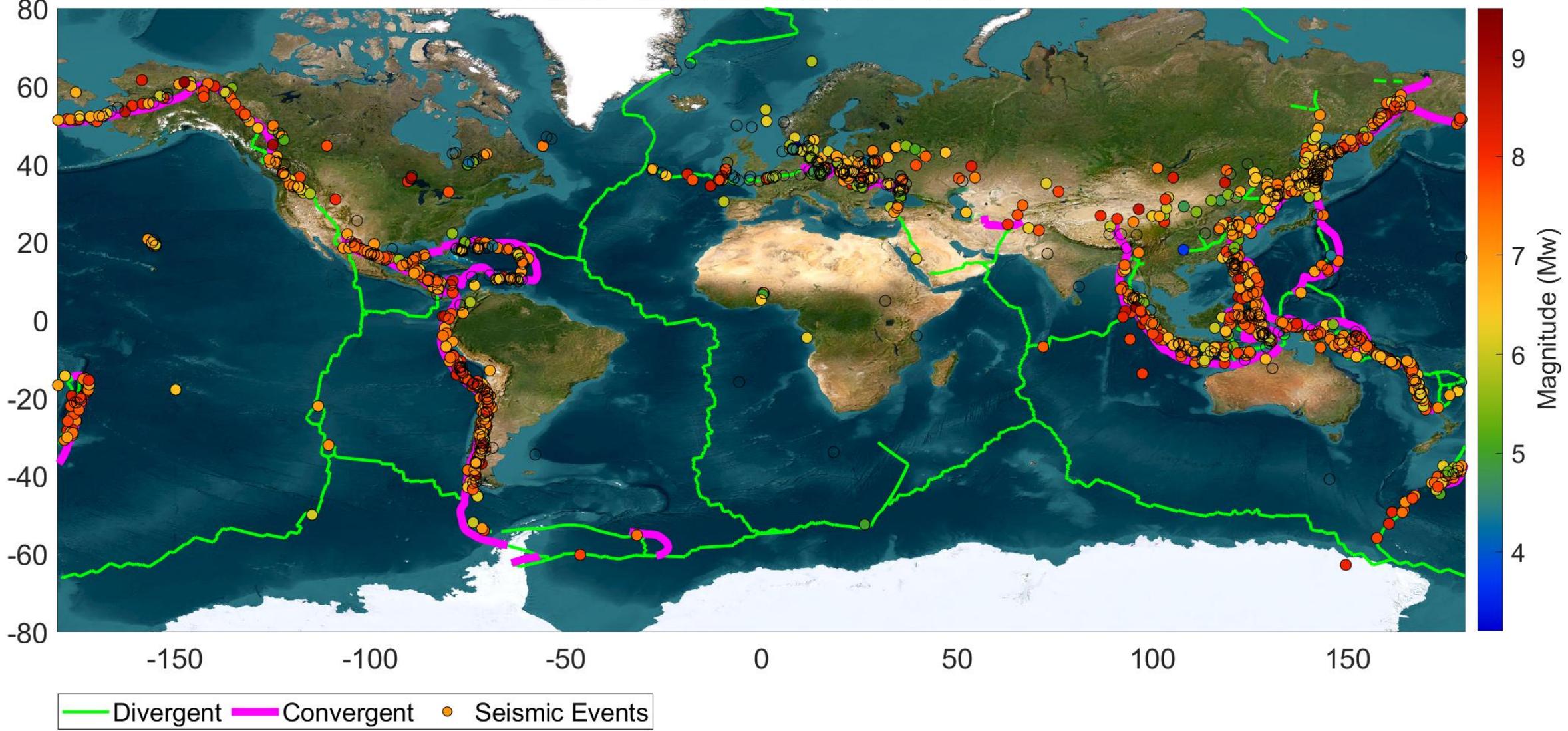
# How does a tsunami occur?





*Tsunami Inundation Modelling and Mapping, INCOIS, Hyderabad, 16-21 March 2026*

### Global Historical Tsunami Events



Reference: [doi:10.7289/V5PN93H7](https://doi.org/10.7289/V5PN93H7)

# Tsunami

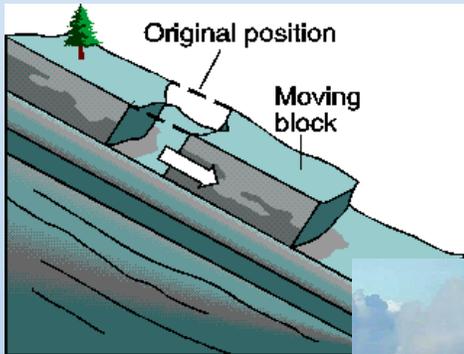
Propagation of disturbance on the ocean free surface under gravitational force from source to coast.

## TSUNAMIS GENERATED BY

- ❖ Earthquakes
- ❖ Landslides
- ❖ Volcanic Explosions
- ☐ Meteo – Tsunamis

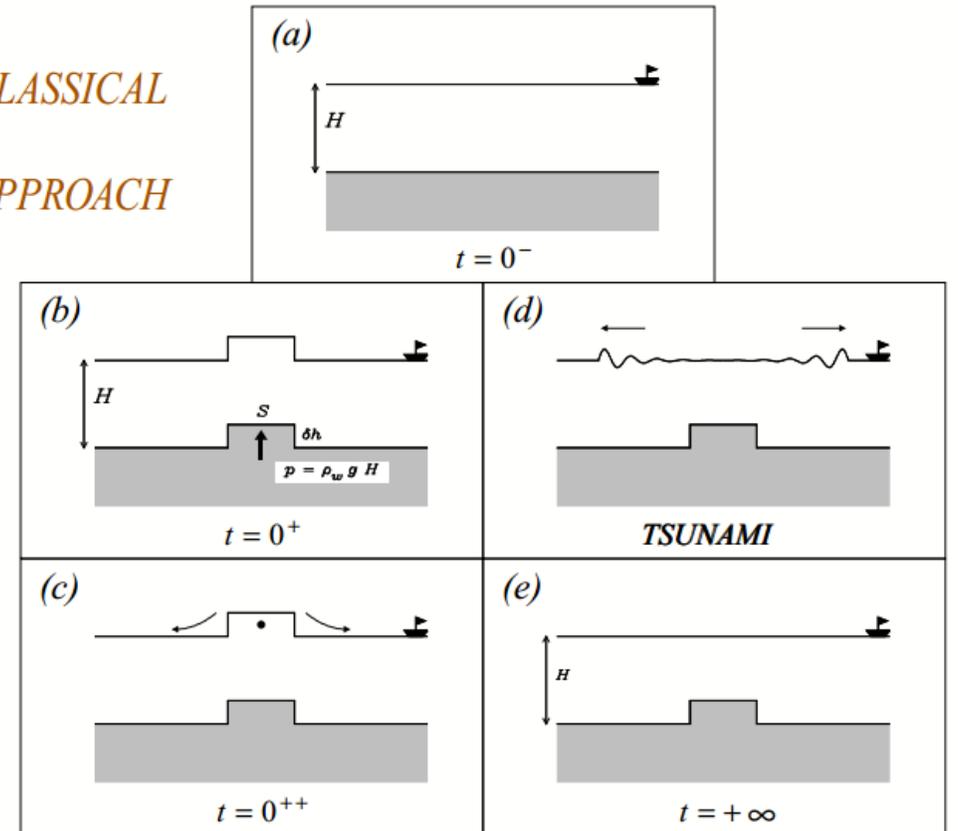
## Tsunami Simulation Steps

1. Generation
2. Propagation
3. Run Up/ Inundation



CLASSICAL

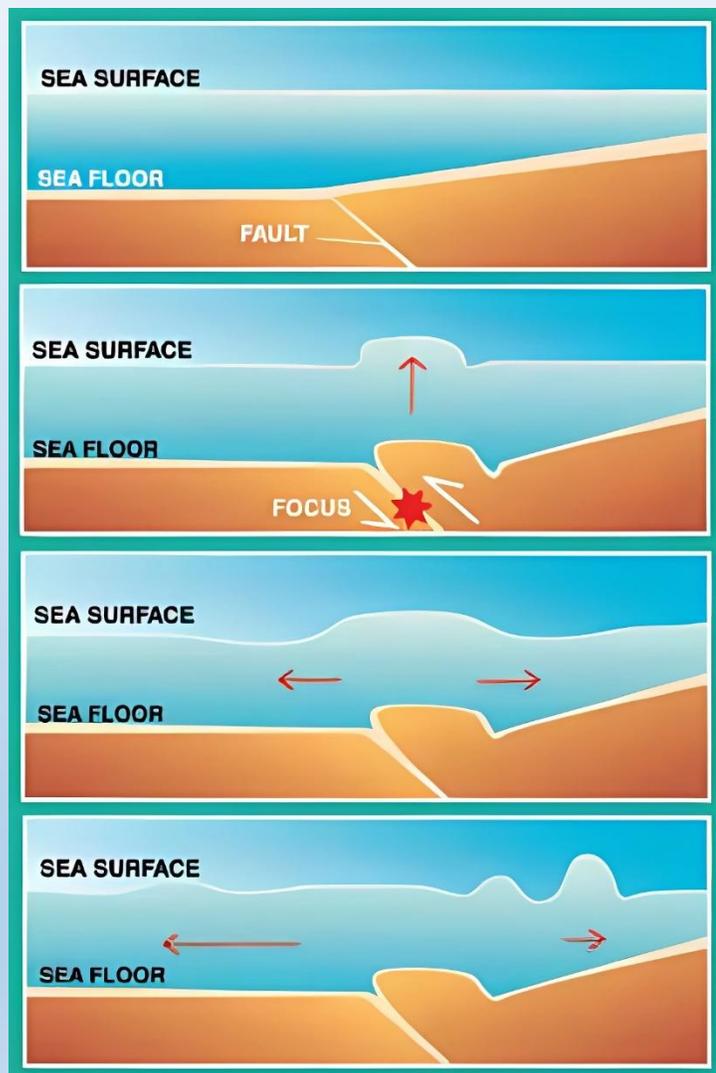
APPROACH



Classical Approach: Source – Prof. Emile A. Okal, Northwestern University, 2017.

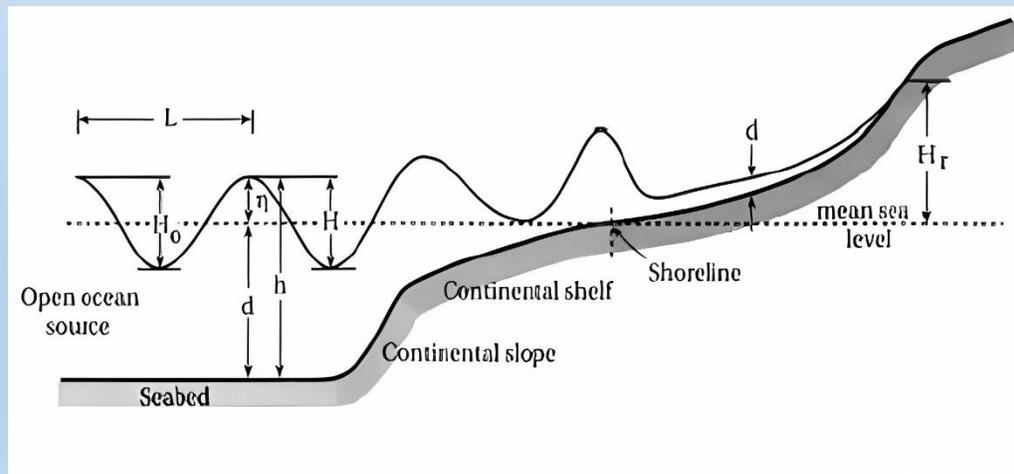
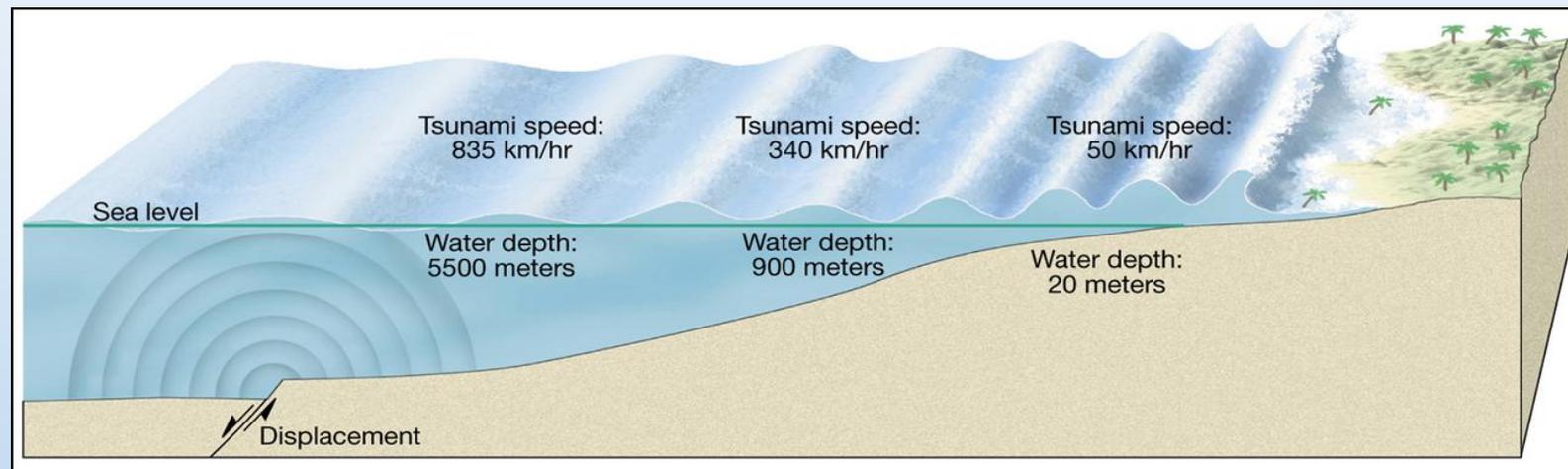
# Tsunami

Propagation of disturbance on the ocean-free surface under gravitational force from source to coast.



Classical approach

←Generation→ ←Propagation→ ←Inundation→



$L$	:	wavelength (m)
$H_0$	:	wave height at deep water (m)
$d$	:	water depth (m)
$\eta$	:	wave elevation (m)
$h$	:	water elevation (m)
$H$	:	wave height (m)
$H_r$	:	wave run-up (m)

Tsunami characteristics and definitions

# Tsunami Characteristics

## Length and Time Period

- Long wavelength (of several 100 km)
- Periods of a few minutes to about an hour

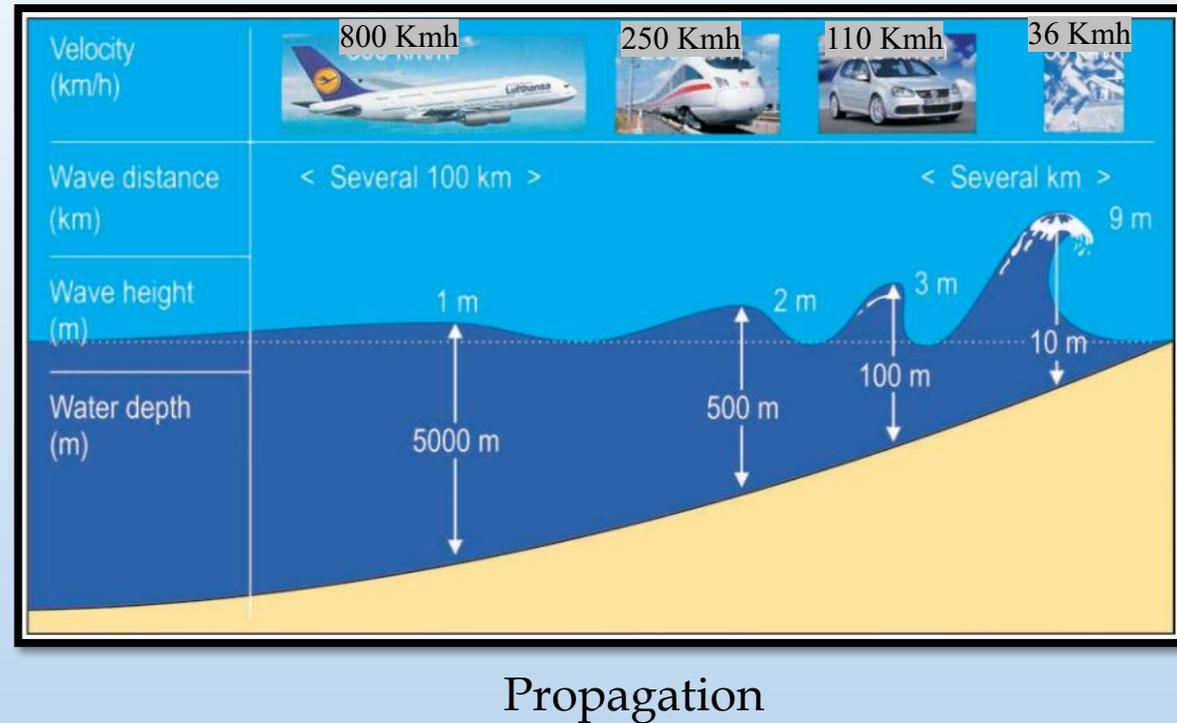
## Speed proportional to the square root of water depth

$$\text{Speed} = \sqrt{gh}$$

- 500 to 1000 km per hour in Deep Ocean
- About 30 km per hour near the shore

## Height of Tsunami Wave

- Less than a meter in the Deep Ocean
- Grows to Tens of meters near the shore



**IN THE DEEP OCEAN** tsunami has a long wavelength, travels fast, and has a small amplitude

As it approaches the shore, it slows. Since energy is conserved, amplitude builds up.

# Governing Equations for tsunami

## The Navier-Stokes Equations

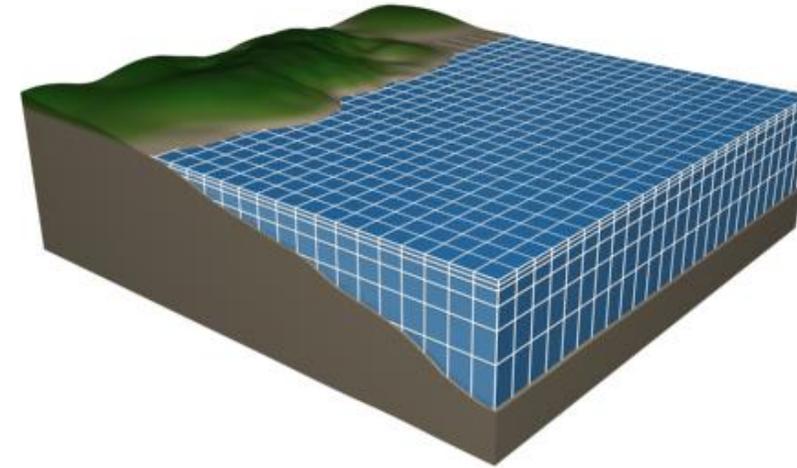
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = \frac{\partial(\tau_{xx} - p)}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z}$$

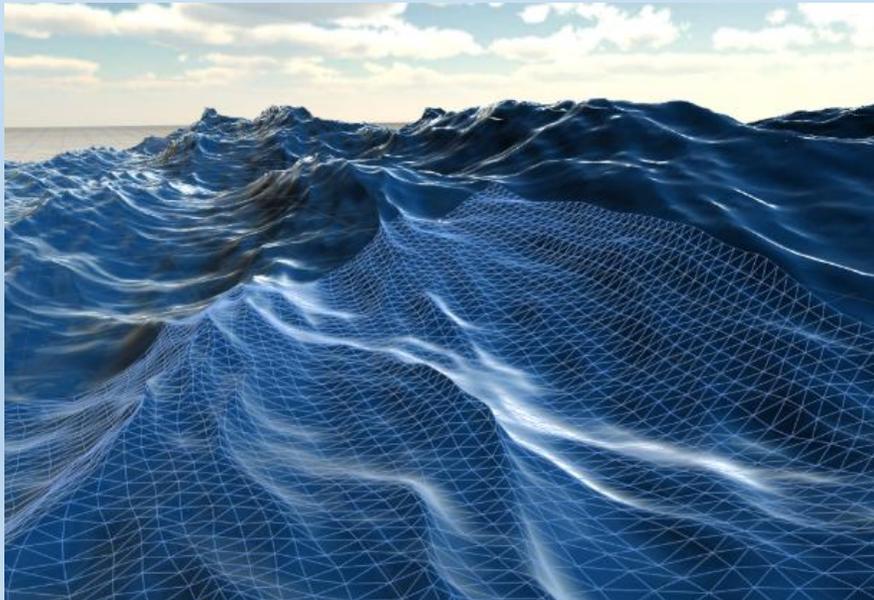
$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial(\tau_{yy} - p)}{\partial y} + \frac{\partial\tau_{yz}}{\partial z}$$

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\rho g + \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial(\tau_{zz} - p)}{\partial z}$$

3-D Grid on the Coastal Ocean



©The COMET Program



## The Shallow Water Equations

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(H\bar{u}) + \frac{\partial}{\partial y}(H\bar{v}) = 0$$

$$\frac{\partial}{\partial t}(H\bar{u}) + \frac{\partial}{\partial x}(H\bar{u}^2) + \frac{\partial}{\partial y}(H\bar{u}\bar{v}) = -gH\frac{\partial\zeta}{\partial x} + \frac{1}{\rho}[\tau_{sx} - \tau_{bx} + F_x]$$

$$\frac{\partial}{\partial t}(H\bar{v}) + \frac{\partial}{\partial x}(H\bar{u}\bar{v}) + \frac{\partial}{\partial y}(H\bar{v}^2) = -gH\frac{\partial\zeta}{\partial y} + \frac{1}{\rho}[\tau_{sy} - \tau_{by} + F_y]$$

## The Shallow water Equations

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M \sqrt{M^2 + N^2} = 0$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{gn^2}{D^{7/3}} N \sqrt{M^2 + N^2} = 0$$

$$\frac{\partial \eta}{\partial t} + \frac{1}{R \cos \theta} \left[ \frac{\partial M}{\partial \lambda} + \frac{\partial}{\partial \theta} (N \cos \theta) \right] = 0$$

$$\frac{\partial M}{\partial t} + \frac{gh}{R \cos \theta} \frac{\partial \eta}{\partial \lambda} = fN$$

$$\frac{\partial N}{\partial t} + \frac{gh}{R} \frac{\partial \eta}{\partial \theta} = -fM$$

$$H_t + (uH)_x + (vH)_y = 0,$$

$$u_t + uu_x + vu_y + gH_x = gD_x$$

$$v_t + uv_x + vv_y + gH_y = gD_y$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial uH}{\partial x} + \frac{\partial vH}{\partial y} = 0$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - fV = -\frac{\partial}{\partial x} \left[ \frac{p_s}{\rho_0} + g\zeta - g(\eta + \gamma) \right] + \frac{\tau_{sx}}{\rho_0 H} - \frac{\tau_{bx}}{\rho_0 H}$$

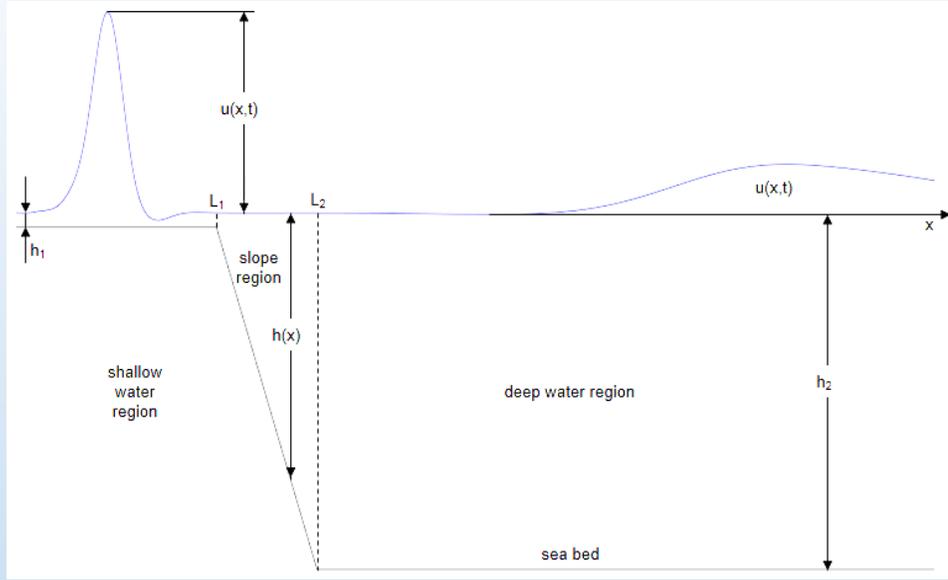
$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + fU = -\frac{\partial}{\partial y} \left[ \frac{p_s}{\rho_0} + g\zeta - g(\eta + \gamma) \right] + \frac{\tau_{sy}}{\rho_0 H} - \frac{\tau_{by}}{\rho_0 H}$$

$$h_t + \frac{(uh)_\lambda + (vh \cos \phi)_\phi}{R \cos \phi} = 0$$

$$u_t + \frac{uu_\lambda}{R \cos \phi} + \frac{vu_\phi}{R} + \frac{gh_\lambda}{R \cos \phi} = \frac{gd_\lambda}{R \cos \phi} + fv$$

$$v_t + \frac{uv_\lambda}{R \cos \phi} + \frac{vv_\phi}{R} + \frac{gh_\phi}{R} = \frac{gd_\phi}{R} - fu$$

# Analytical 1D PDE of Tsunami Model



Tsunami Model: Wave Equation Solution  
Governing PDE (linear 1D water theory)

$$u_{tt} = g (h(x) u_x)_x$$

- $u(x, t)$ : free surface elevation -  $g = 9.81 \text{ m/s}^2$ : gravity -  $h(x)$ : water depth
- Depth Profile (slope region  $L_1 \leq x \leq L_2$ )

$$h(x) = \frac{H}{L} x, \quad h_1 = \text{depthratio} \cdot H, \quad h_2 = H$$

- $H$ : deep water depth -  $L$ : slope length -  $\text{depthratio} = h_1/h_2$ : shallow/deep depth ratio
- Fourier Mode Decomposition ( $u(x, t) = U(x, \omega) e^{i\omega t}$ )
- Deep water ( $h = h_2$ ):

$$u_2(x, t) = e^{i\omega(t+x/c_2)} + R(\omega) e^{i\omega(t-x/c_2)}, \quad c_2 = \sqrt{gh_2}$$

Shallow water ( $h = h_1$ ):

$$u_1(x, t) = T(\omega) e^{i\omega(t+x/c_1)}, \quad c_1 = \sqrt{gh_1}$$

Slope region:

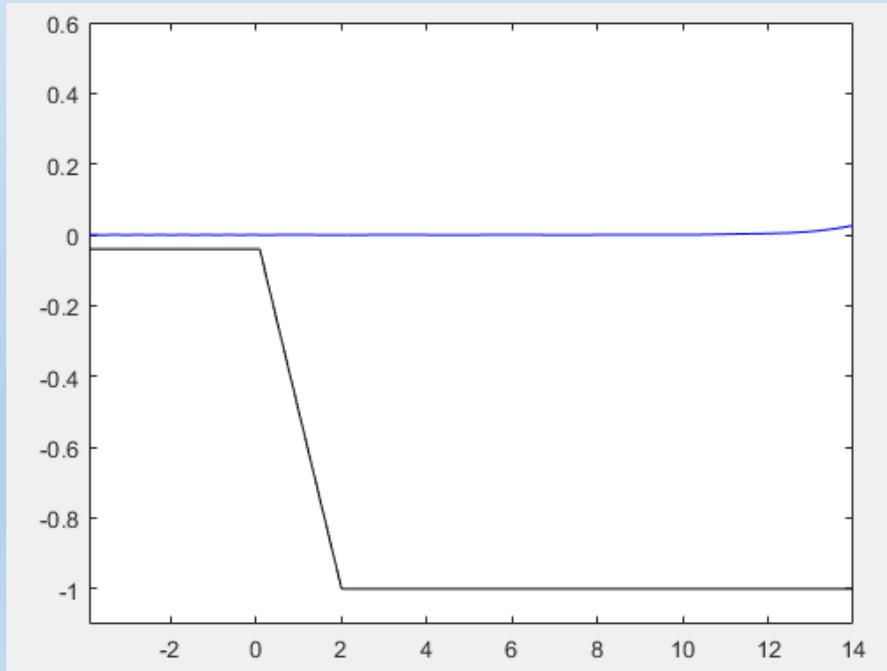
$$U(x, \omega) = C_1 J_0 \left( 2\omega \sqrt{\frac{Lx}{gH}} \right) + C_2 Y_0 \left( 2\omega \sqrt{\frac{Lx}{gH}} \right)$$

$J_0, Y_0$ : Bessel functions of order zero.

Boundary Matching (continuity of  $u$  and  $u_x$  at  $x = L_1, L_2$ ) determines constants  $C_1, C_2, R(\omega), T(\omega)$ .  
Low Frequency Limits (dominant tsunami energy)

$$\lim_{\omega \rightarrow 0} R = \frac{1 - \sqrt{\text{depthratio}}}{1 + \sqrt{\text{depthratio}}}, \quad \lim_{\omega \rightarrow 0} T = \frac{2}{1 + \sqrt{\text{depthratio}}}$$

Full Wave Field obtained via inverse FFT of frequency domain solutions.



# Real-world Problems

Structural Analysis, Thermal Analysis, Fluid Analysis

ODE/ PDE/ Matrices

## 3. Numerical Methods/ Numerical Simulations

- Finite Difference Method (FDM)
- Finite Element Method (FEM)
- Finite Volume Method (FVM)
- Boundary Element Method (BEM)
- Meshless Method (SPH)
- Finite Particle Method (FPM)
- Monte Carlo Methods (MC)

## 2. Experimental Analysis

## 1. Analytical Solutions

# A General Procedure of Numerical Methods

## Preprocessing

- Define the geometric domain of the problem of interest
- Define the element type based on the deformation
- Define the geometric properties of the elements
- Define the material properties
- Define the nodal connectivity (constitutes a mesh)
- Define the boundary conditions

## Mathematical solution

- Compute the unknown variables

## Postprocessing

- Visualizing the results
- Sorting, plotting, printing

# Programming/ Coding/ Packages

**C, C++**

**Fortran**

**Python**

**R**

**Julia**

**Mathematica**

**MATLAB**

**Octave**

**Scilab**

**ABAQUS**

**ANSYS**

**Cast3M**

**Fluent**

**Open Foam**

# Methods to solve the Shallow water Equations

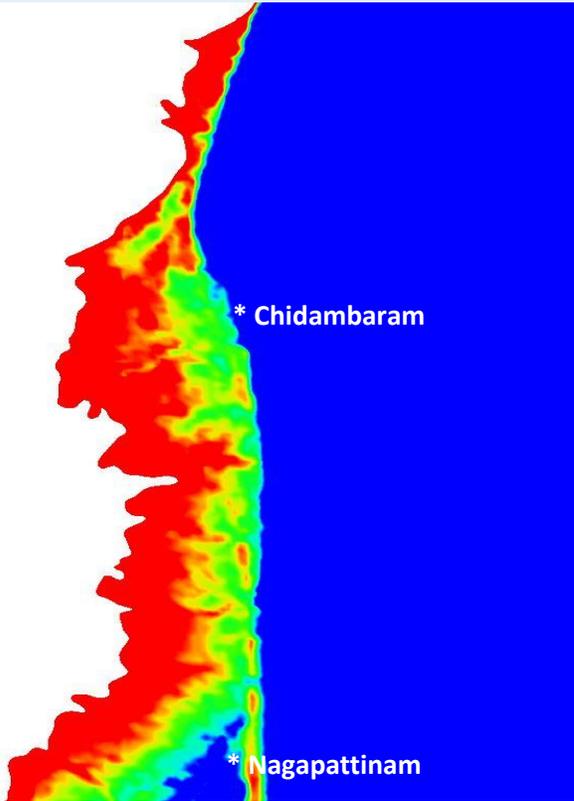
1	<b>Analytical solutions</b> Closed form solution. Not possible for all the cases.
2	<b>Finite Difference Method (FDM)</b> Differential Equations are discretized using Taylors series and time integrated Ex: comMIT, TUNAMI, COMCOT, MOST
3	<b>Finite Element Method (FEM)</b> Subdivides domain into smaller parts called finite elements and are then assembled into the main domain Ex: ADCIRC, CAST3M, (ABAQUS, ANSYS, COMSOL)
4	<b>Finite Volume Method (FVM)</b> Volume integrals with a divergence term are converted to surface integrals, using the divergence theorem, and fluxes are evaluated at the surfaces Ex: ANUGA, CLAWPACK, FVCOM, HySEA, OPEN-FOAM, (FLUENT)
5	<b>Meshfree Methods</b> The differentials in are converted into a summation formula using kernel functions which operates on nearby data points Ex: Smoothed Particle Hydrodynamics (SPH)

# Tsunami Modelling and Coastal Inundation Requirements

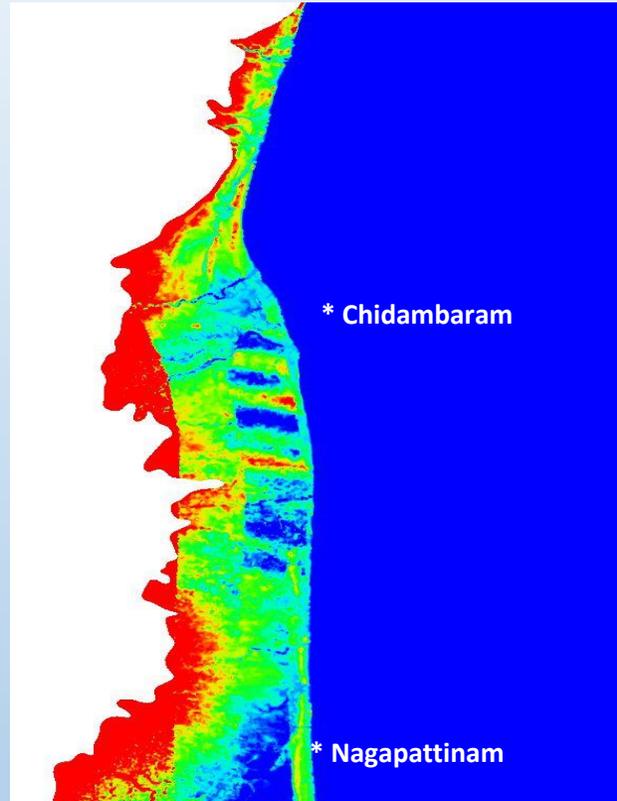
<b>Model</b>	<b>A high-resolution, physics-based SWE solver with flooding and drying capabilities Supports parallel computing</b>
<b>Preprocessing</b>	
	High resolution mesh/ grid of region of interest
<b>Generation</b>	Seismic fault parameters Accurate Initial deformation Mansinha and Smiley; Okada model
<b>Solving</b>	
<b>Propagation</b>	A high-resolution water depth (bathymetry)
<b>Inundation</b>	A high-resolution land elevation (topography) elevation
<b>Postprocessing</b>	
<b>Analysis</b>	Wave heights, travel times, inundated area (Plotting tools) (Observations)

# Propagation and Coastal Inundation Requirements (preprocessing)

## Importance of bathymetry and inland topography

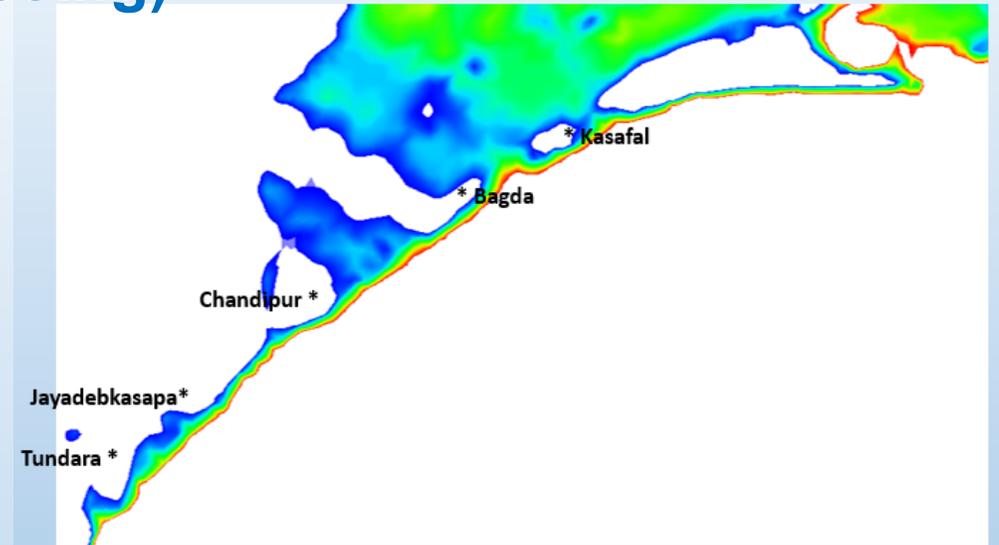


GEBCO 30 arc sec

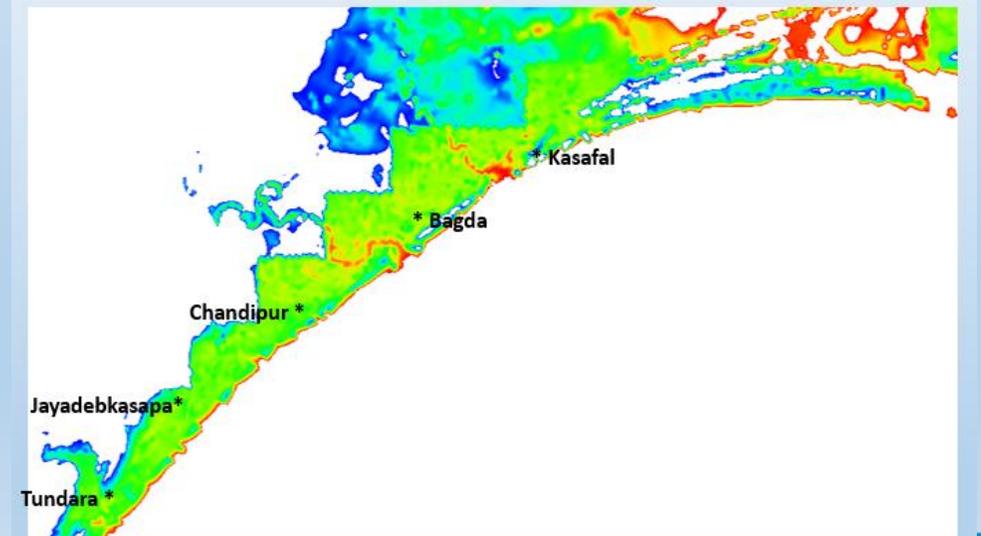


ALTM+CARTOSAT+SRTM

GEBCO 30 arc sec topography



SRTM+CARTOSAT+ALTM topography



# GEBCO (General Bathymetric Chart of the Oceans)

# SRTM (Shuttle Radar Topography Mission)

# Tsunami Generation

1. Formation of an initial disturbance of the ocean surface due to the earthquake-triggered deformation of the sea floor
2. Initial water surface disturbance - A long gravity wave radiating from the earthquake source
3. The basic parameters required to generate initial deformation:

Earthquake Hypocenter	
1	Moment magnitude of the Earthquake ( $M_w$ )
2	Location (Latitude, Longitude) in degrees
3	Focal depth (km)
Focal Mechanism	
4	Strike angle ( $0^\circ$ to $360^\circ$ )
5	Dip angle ( $0^\circ$ to $90^\circ$ )
6	Slip/ Rake angle ( $0^\circ$ to $\pm 180^\circ$ )
Fault geometry	
7	Slip (m)
8	Fault Length (km)
9	Fault Width (km)

$$\log_{10}(M_0) = 1.5M_w + 9.05$$

Strike- Along the plate boundary or parallel to the coast, Dip 45 degrees, Rake 90 degrees

Dip =  $90^\circ$  Rake =  $0^\circ$  :: left-lateral strike-slip

Dip =  $90^\circ$  Rake =  $180^\circ$  :: right lateral strike slip

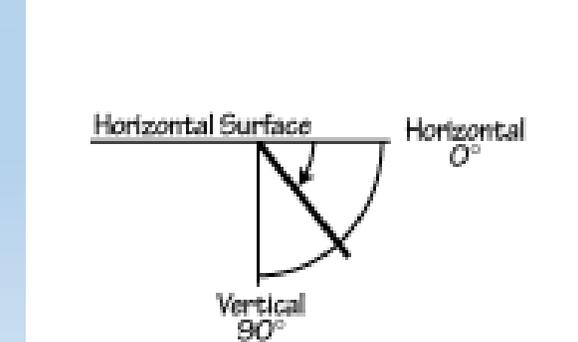
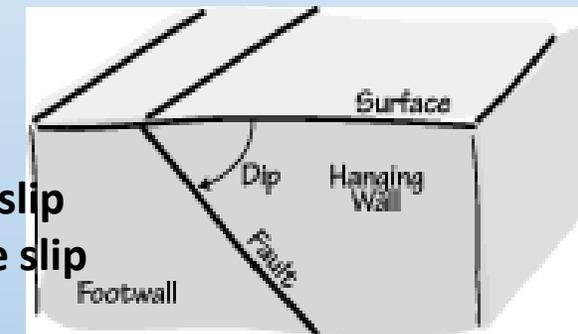
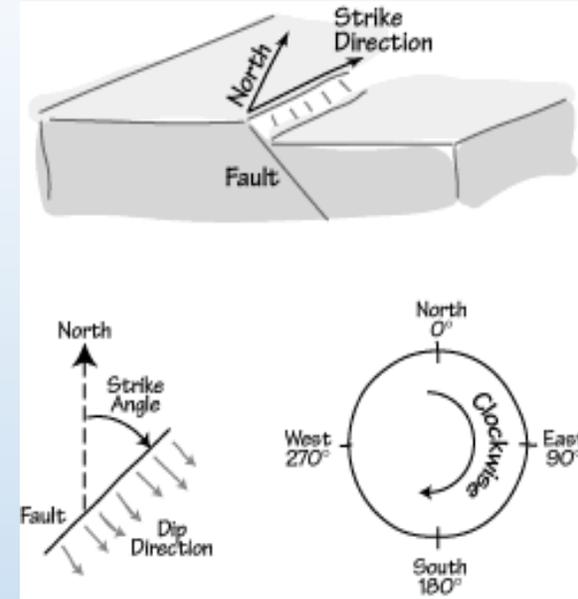
Dip =  $45^\circ$  Rake =  $90^\circ$  :: reverse fault

Dip =  $45^\circ$  Rake =  $-90^\circ$  :: normal fault

$$M_0 = \mu AD$$

$$\log_{10} L = a + bM_w$$

$$\log_{10} W = a + bM_w$$



# Governing Equations

## Non-linear Shallow water equations

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M \sqrt{M^2 + N^2} = 0$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{gn^2}{D^{7/3}} N \sqrt{M^2 + N^2} = 0$$



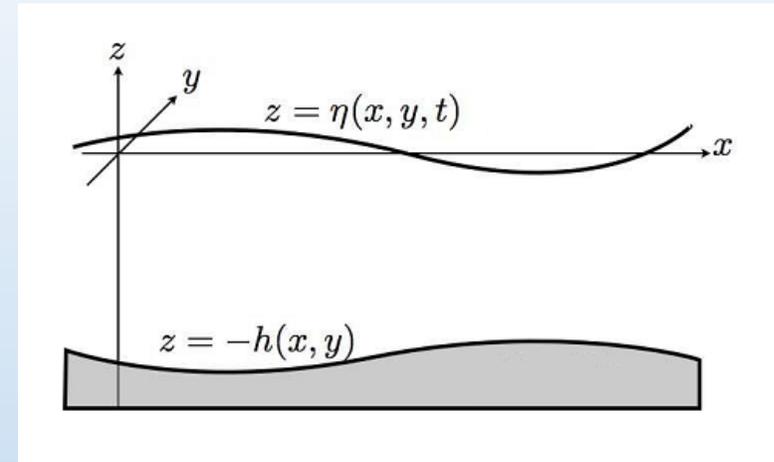
## Linear Shallow water equations

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial M}{\partial t} + gD \frac{\partial \eta}{\partial x} = 0$$

$$\frac{\partial N}{\partial t} + gD \frac{\partial \eta}{\partial y} = 0$$

$$\lambda \gg h$$



$\eta$  → Wave height

$M, N$  → discharge fluxes in x and y directions

$D$  → Total water depth

$n$  → Mannings roughness coefficient

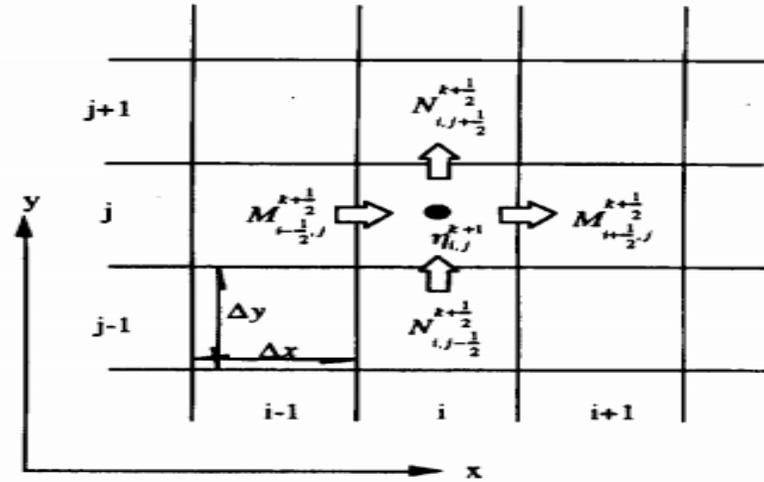
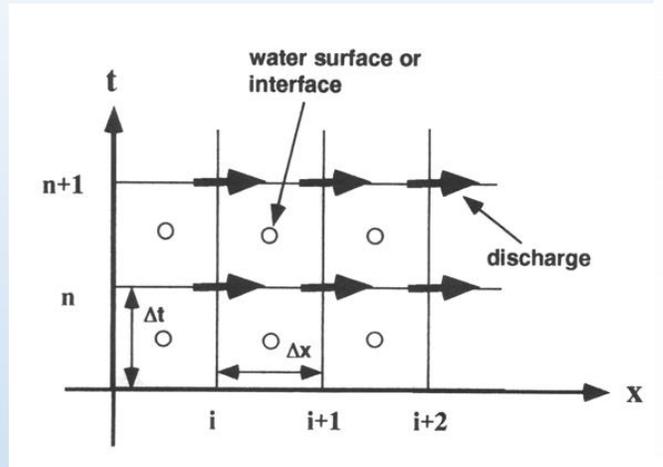
$M = u(h + \eta) = uD$   $N = v(h + \eta) = vD$

$$n = \sqrt{\frac{fD^{1/3}}{2g}}$$

$u, v$  → velocity in x and y directions

$f$  → friction coefficient

# Numerical Methodology



1. Initial boundary value problem
2. Equations are solved by employing finite difference method (FDM)
3. The staggered leap-frog scheme (Shuto, Goto, Imamura, (1990)) is used to solve the governing equations.
4. At every time step, wave is propagated by calculating the water surface elevations and water velocities throughout the domain.
5. For stability the time step and grid size should be selected properly and they should obey CFL condition

$$\frac{\nabla x}{\nabla t} = \sqrt{2gh_{\max}}$$

# Finite Difference Scheme

$$\frac{\partial \eta}{\partial t} = \frac{1}{\Delta t} [\eta_{i,j}^{k+1} - \eta_{i,j}^k]; \frac{\partial M}{\partial x} = \frac{1}{\Delta x} \left[ M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} - M_{i-\frac{1}{2},j}^{k+\frac{1}{2}} \right]; \frac{\partial N}{\partial y} = \frac{1}{\Delta y} \left[ N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} - N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}} \right]$$

$$\eta_{i,j}^{k+1} = \eta_{i,j}^k - \frac{\Delta t}{\Delta x} \left[ M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} - M_{i-\frac{1}{2},j}^{k+\frac{1}{2}} \right] - \frac{\Delta t}{\Delta y} \left[ N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} - N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}} \right] \rightarrow \text{Wave height}$$

$$M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} = M_{i+\frac{1}{2},j}^{k-\frac{1}{2}} - gD_{i+\frac{1}{2},j}^k \frac{\Delta t}{\Delta x} [\eta_{i+1,j}^k - \eta_{i,j}^k]$$

→ Discharge flux along x

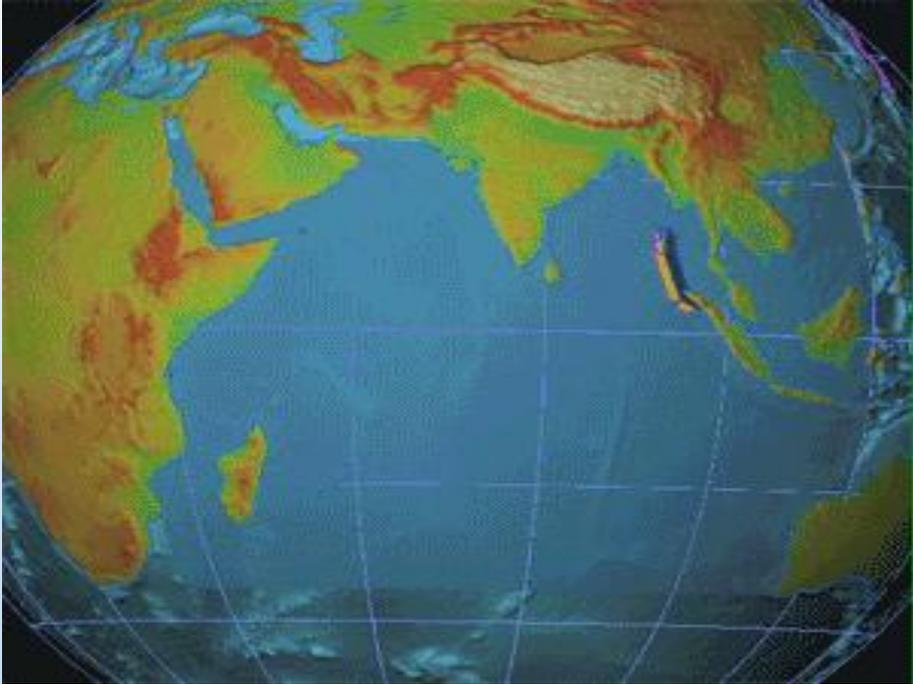
$$D_{i+\frac{1}{2},j}^k = h_{i+\frac{1}{2},j} - \eta_{i+\frac{1}{2},j}^k = h_{i+\frac{1}{2},j} + \frac{1}{2} [\eta_{i+1,j}^k + \eta_{i,j}^k]$$

$$N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} = N_{i,j+\frac{1}{2}}^{k-\frac{1}{2}} - gD_{i,j+\frac{1}{2}}^k \frac{\Delta t}{\Delta y} [\eta_{i,j+1}^k + \eta_{i,j}^k]$$

→ Discharge flux along y

$$D_{i,j+\frac{1}{2}}^k = h_{i,j+\frac{1}{2}} + \eta_{i,j+\frac{1}{2}}^k = h_{i,j+\frac{1}{2}} + \frac{1}{2} [\eta_{i,j+1}^k + \eta_{i,j}^k]$$

# Indian Ocean Tsunami of December 26, 2004

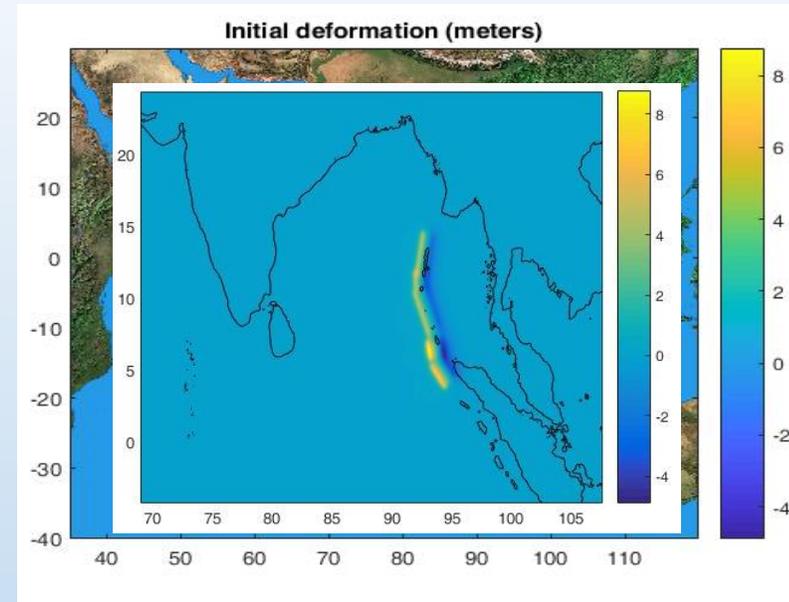
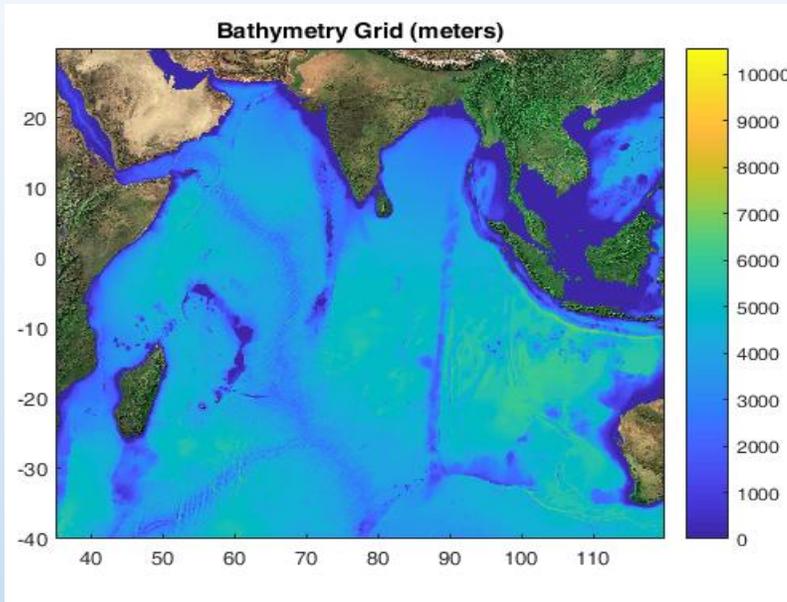


- The worst tsunami in recorded history on December 26, 2004
- Magnitude 9.3 (second strongest earthquake ever recorded on a seismograph)
- Lasted 10 minutes (longest lasting earthquake in history)
- 229,866 confirmed dead, which includes 42,883 missing and never accounted for
- More than \$7 billion dollars damage

## Reasons for huge loss.....

- Many nations in the Indian Ocean did not even recognize the word “tsunami”
- None had tsunami preparedness programs in place
- Absence of a Tsunami Early Warning System (TEWS) in India
- Ignorance of the natural signs of a tsunami led to inappropriate actions

# Generation/Initial deformation

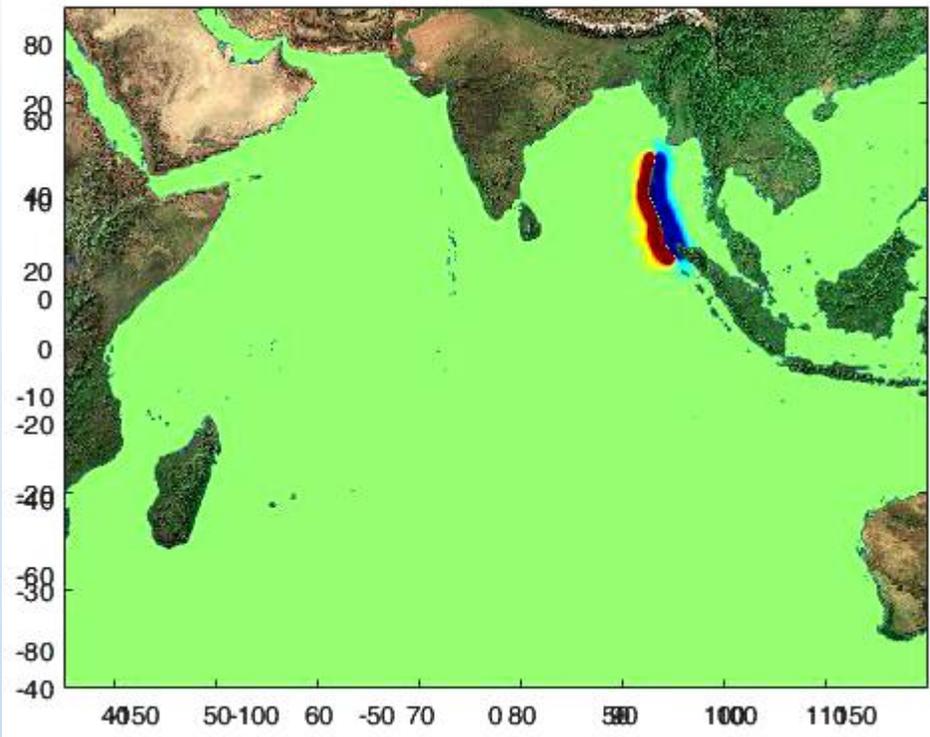


Computed initial deformation due to December 26, 2004, Great Sumatra Earthquake

Parameters	Segment 1	Segment 2	Segment 3	Segment 4	Segment 5
$x_o$ (longitude)	94.57	93.90	93.21	92.60	92.87
$y_o$ (latitude)	3.83	5.22	7.41	9.70	11.70
$d$ (km)	25	25	25	25	25
$\varphi$ (degrees)	323	348	338	356	10
$\lambda$ (degrees)	90	90	90	90	90
$\delta$ (degrees)	12	12	12	12	12
$\Delta$ (m)	18	23	12	12	12
$L$ (km)	220	150	390	150	350
$W$ (km)	130	130	120	95	95

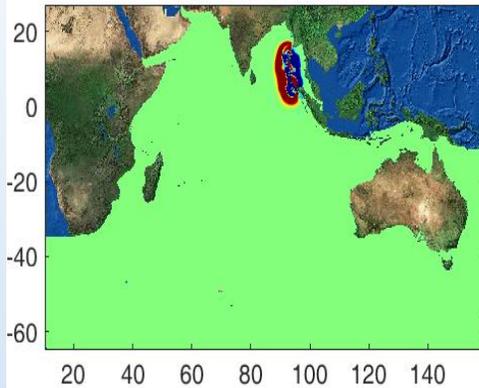
Earthquake fault parameters of the December 26, 2004, Great Sumatra Earthquake were used to compute initial deformation (Source: Grilli et al., 2007)

0 hours 0 mins

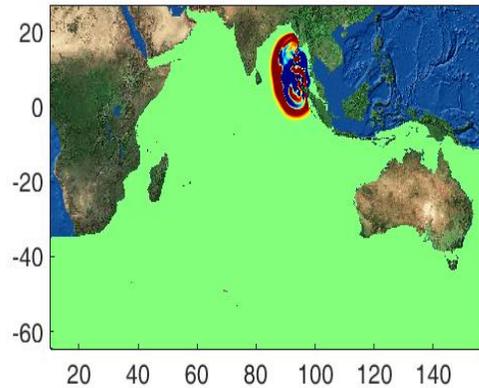


# Propagation

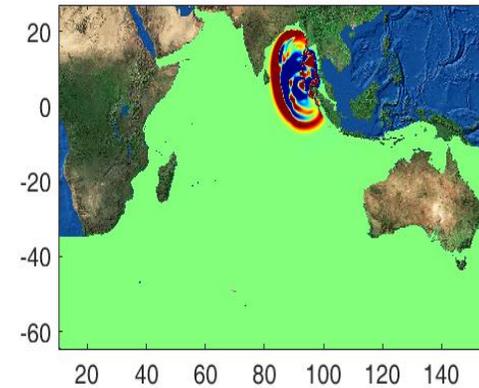
0 hours 30 mins



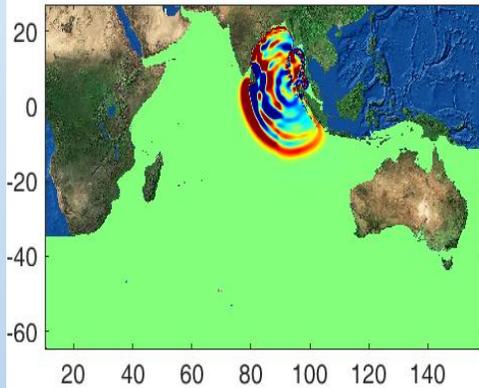
1 hours 0 mins



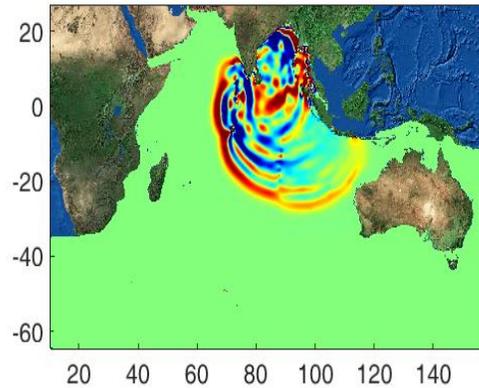
1 hours 30 mins



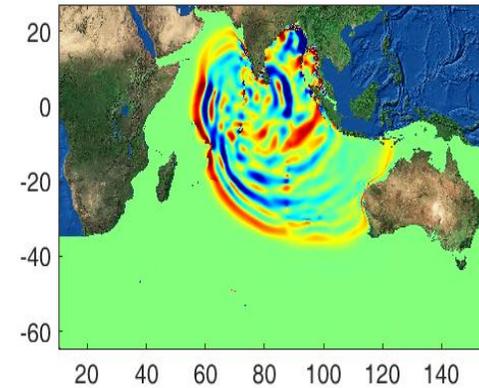
2 hours 30 mins



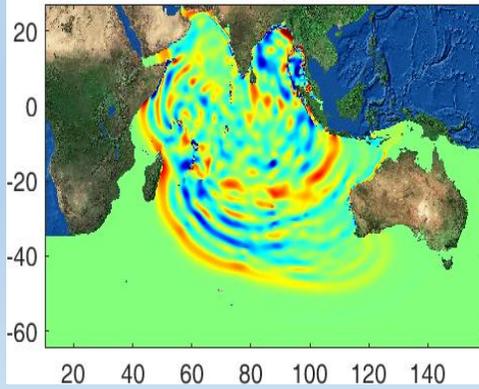
4 hours 30 mins



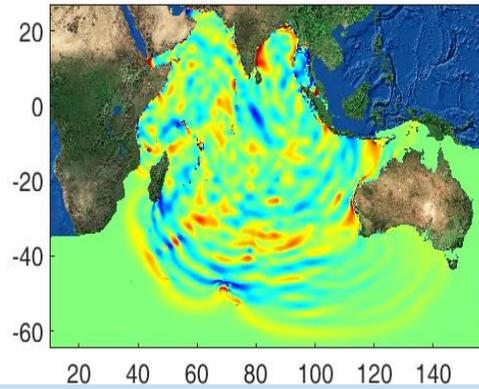
6 hours 0 mins



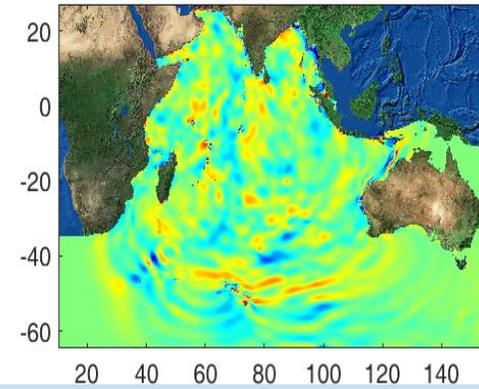
8 hours 0 mins



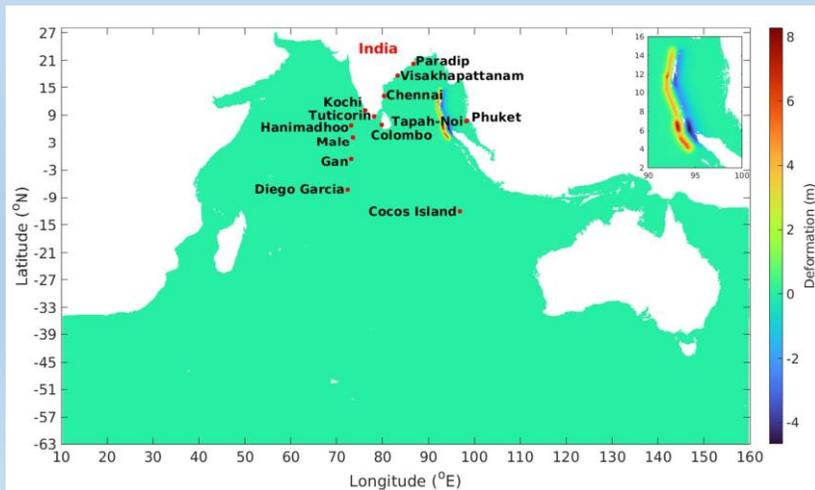
10 hours 0 mins

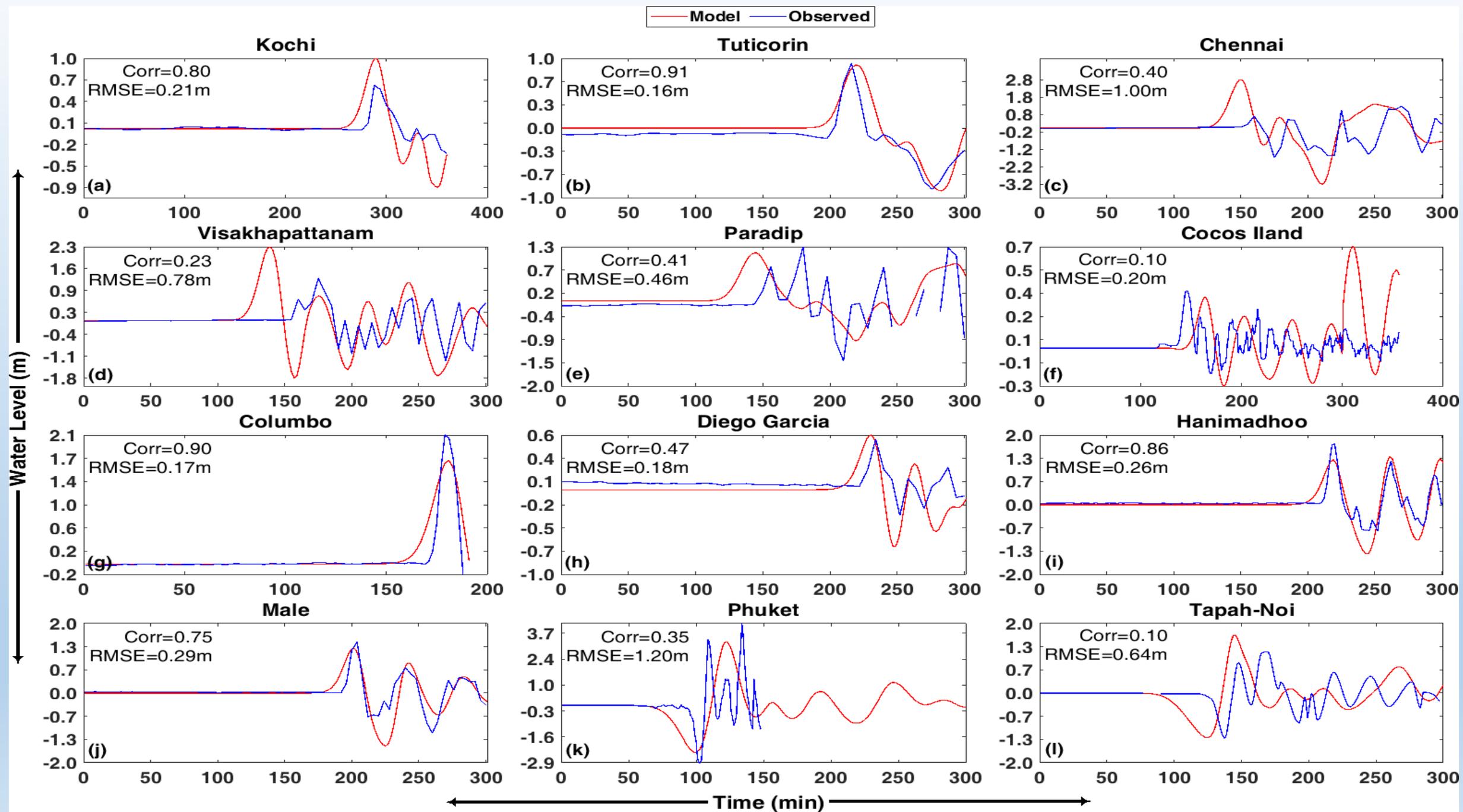


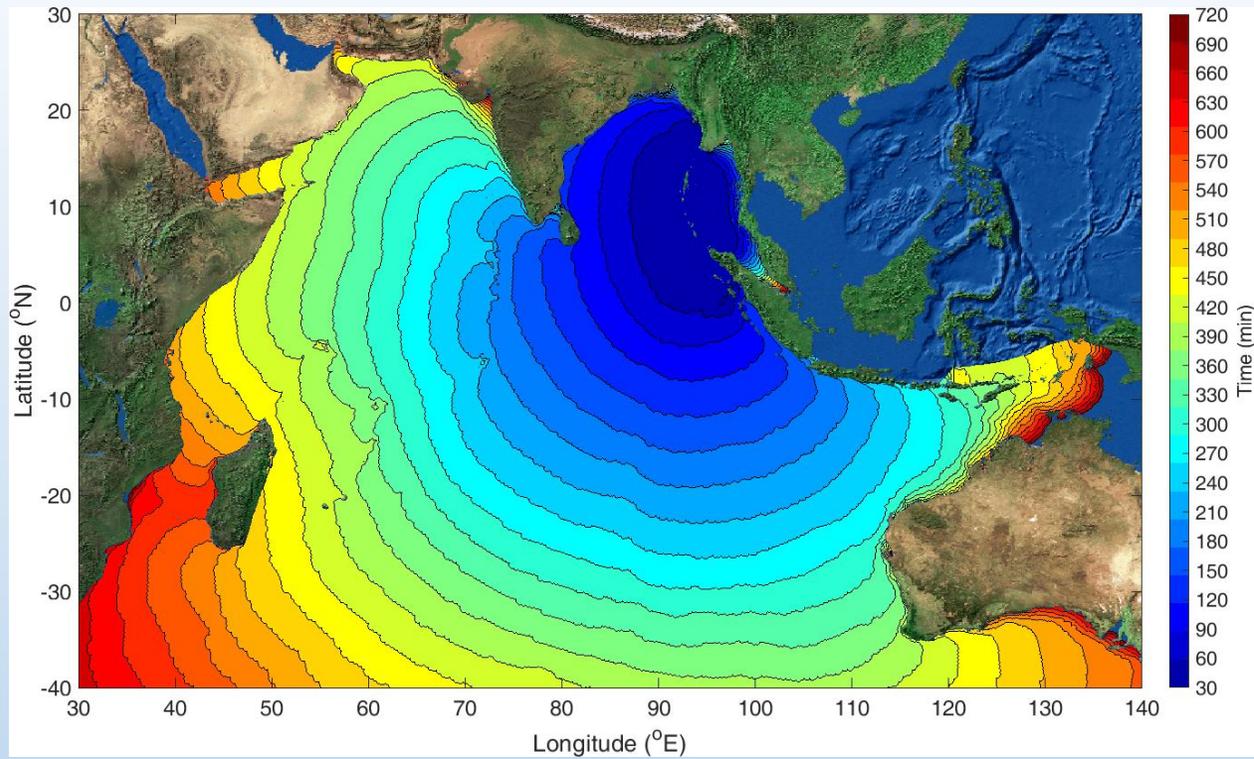
12 hours 0 mins



## Wave propagation Animation

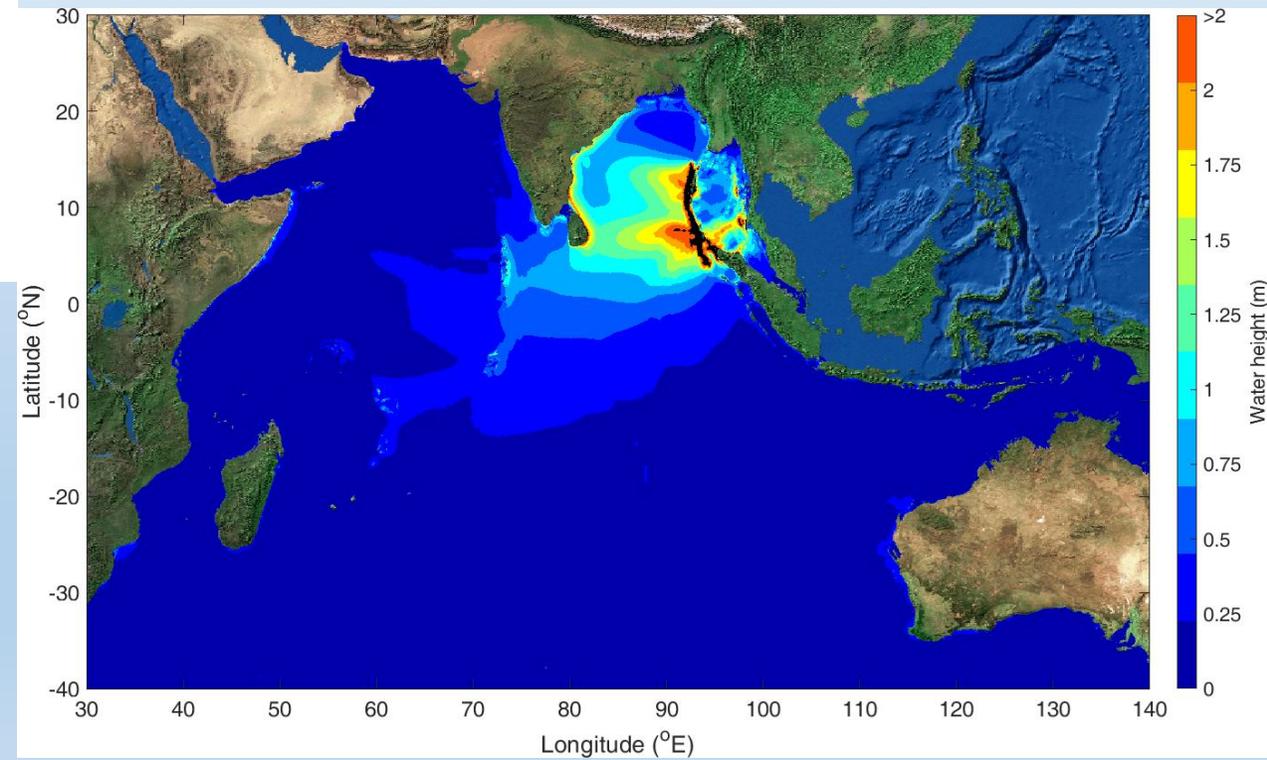


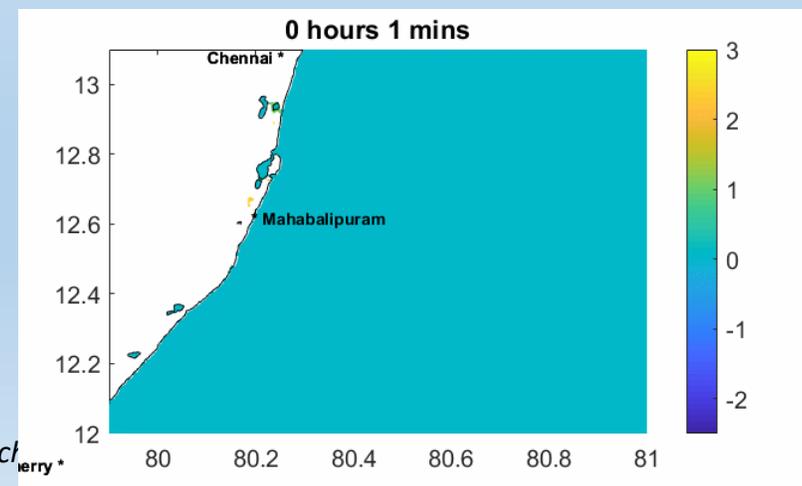
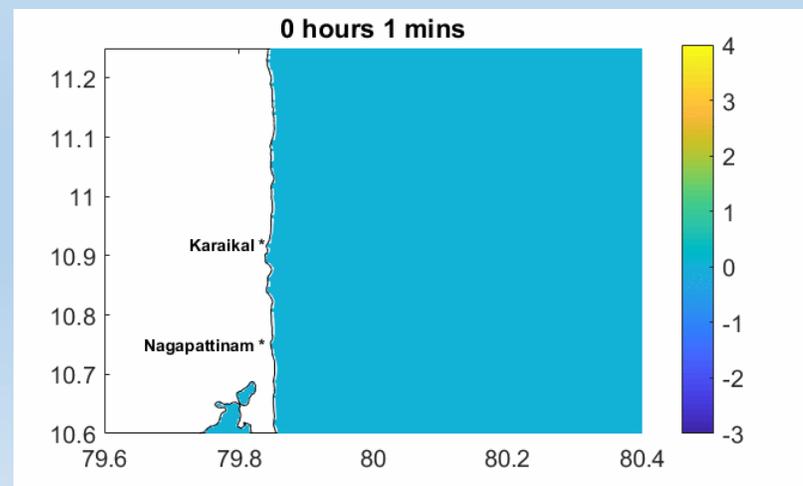
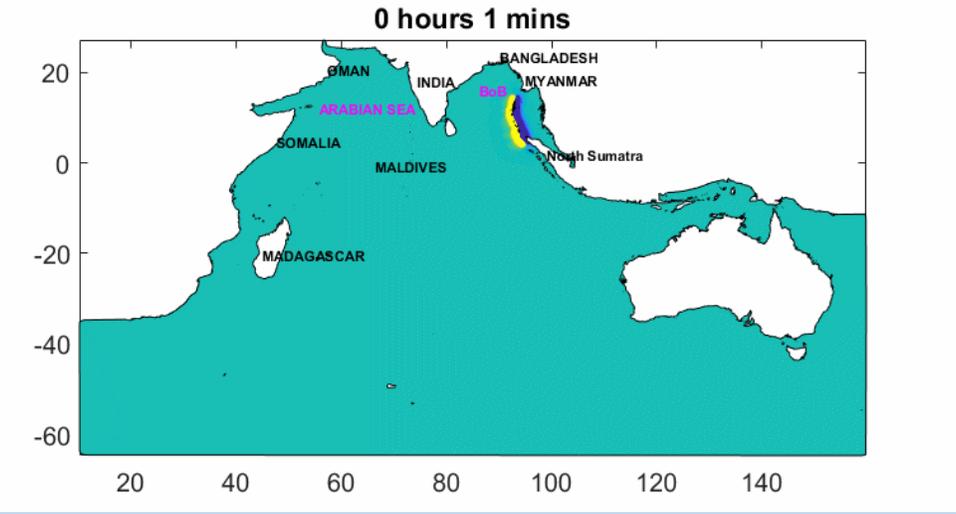
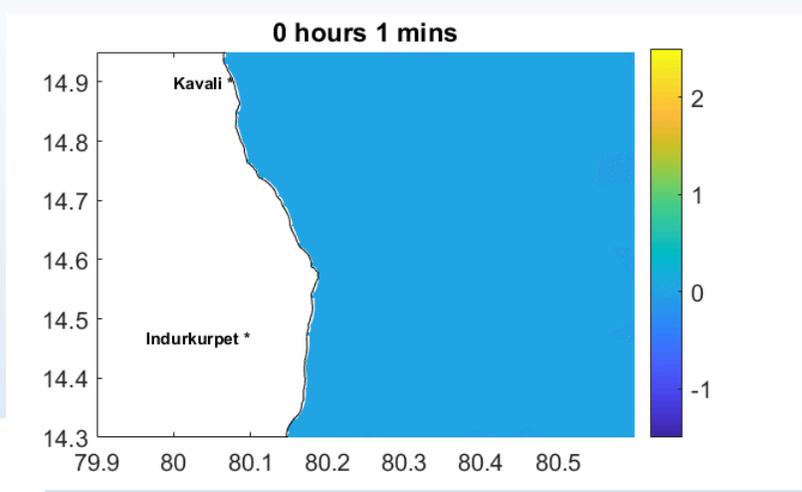
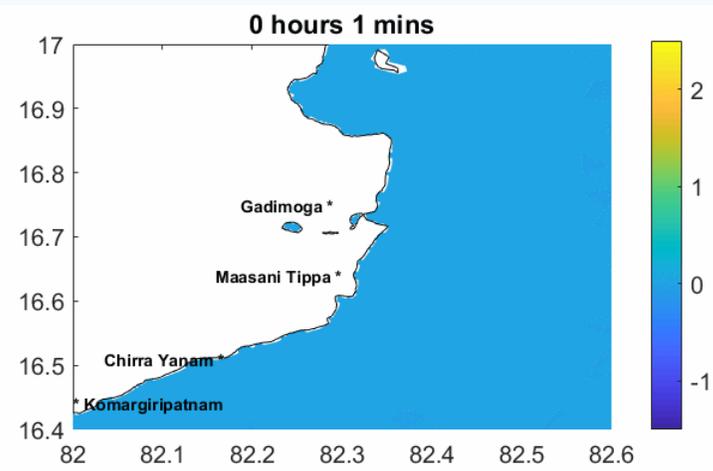




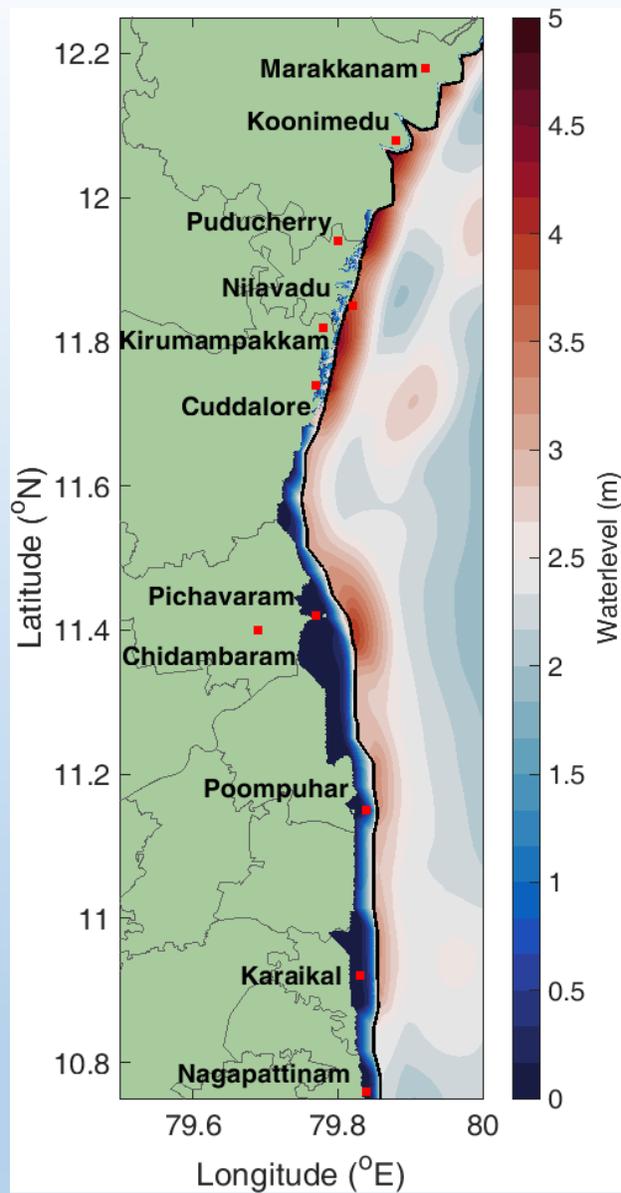
**Travel time contours of the tsunami wave associated with 26th December 2004 Sumatra Earthquake.**

**Directivity Plot (Wave height in meters)**

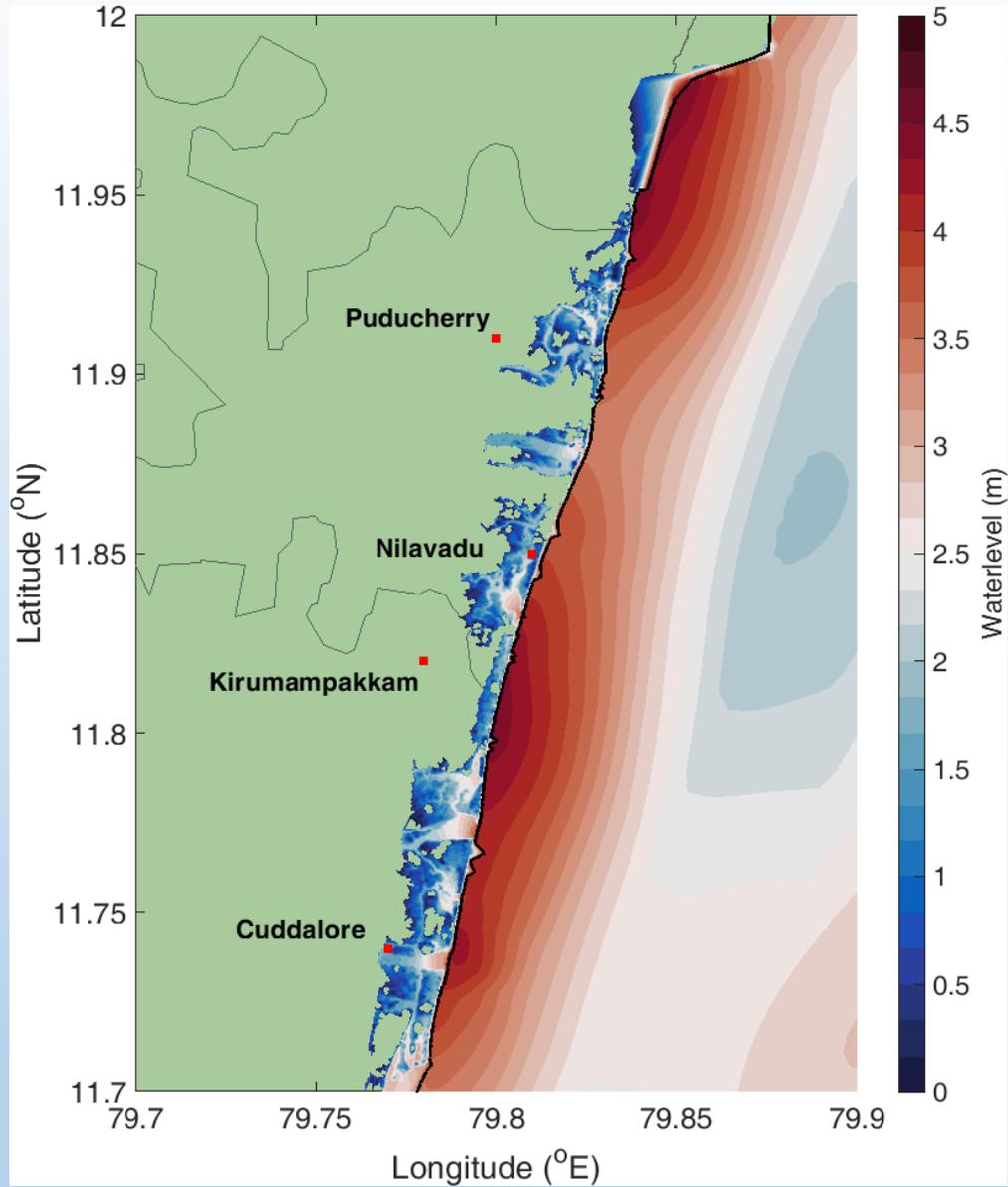




Hyderabad, 16-21 March



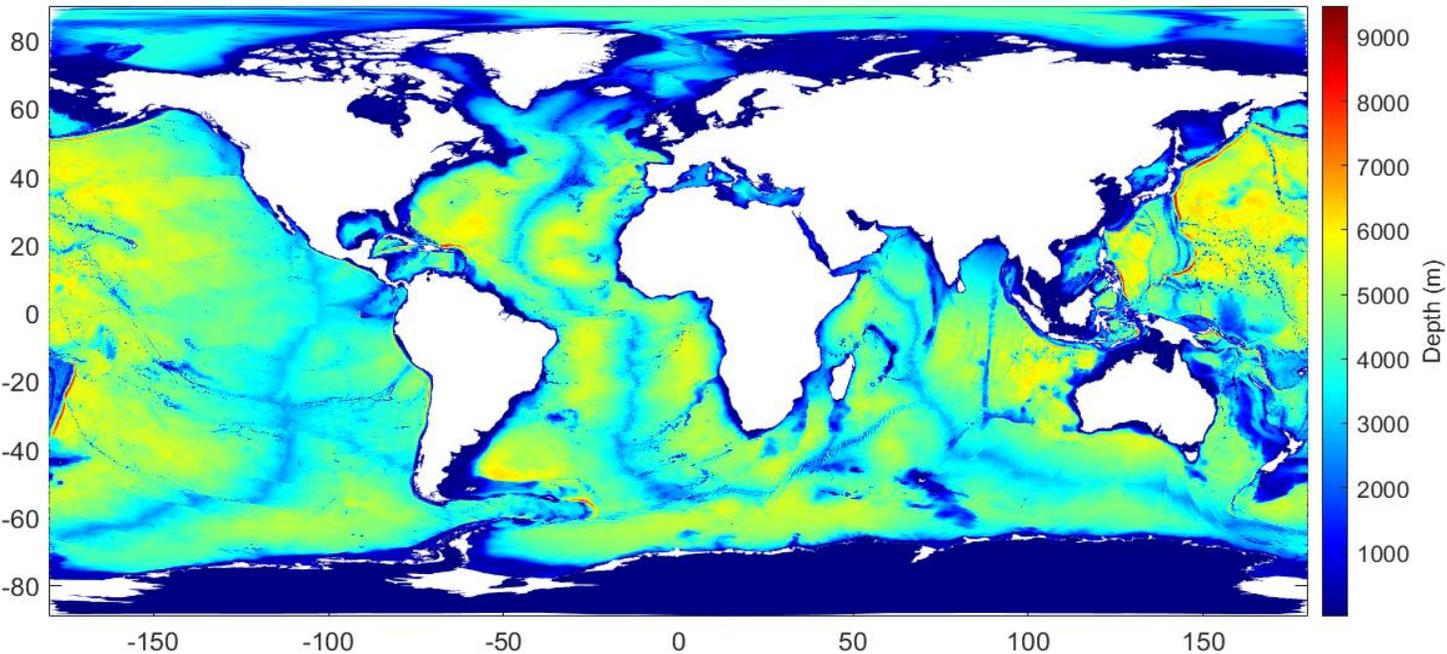
**Spatial depiction of inundation extent along the coastal region of southern Tamil Nadu**



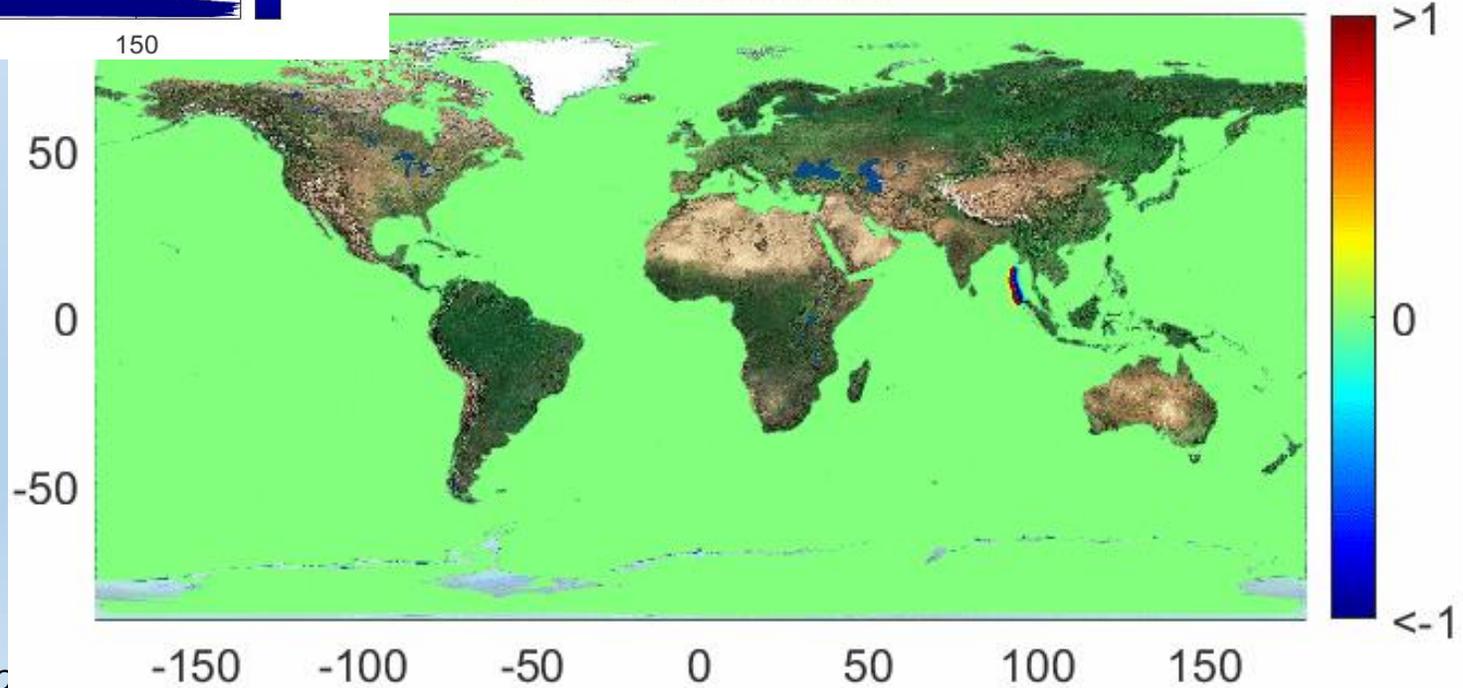
**Enlarged portion of depicting inundation where the merged bathymetry and topography data were used**

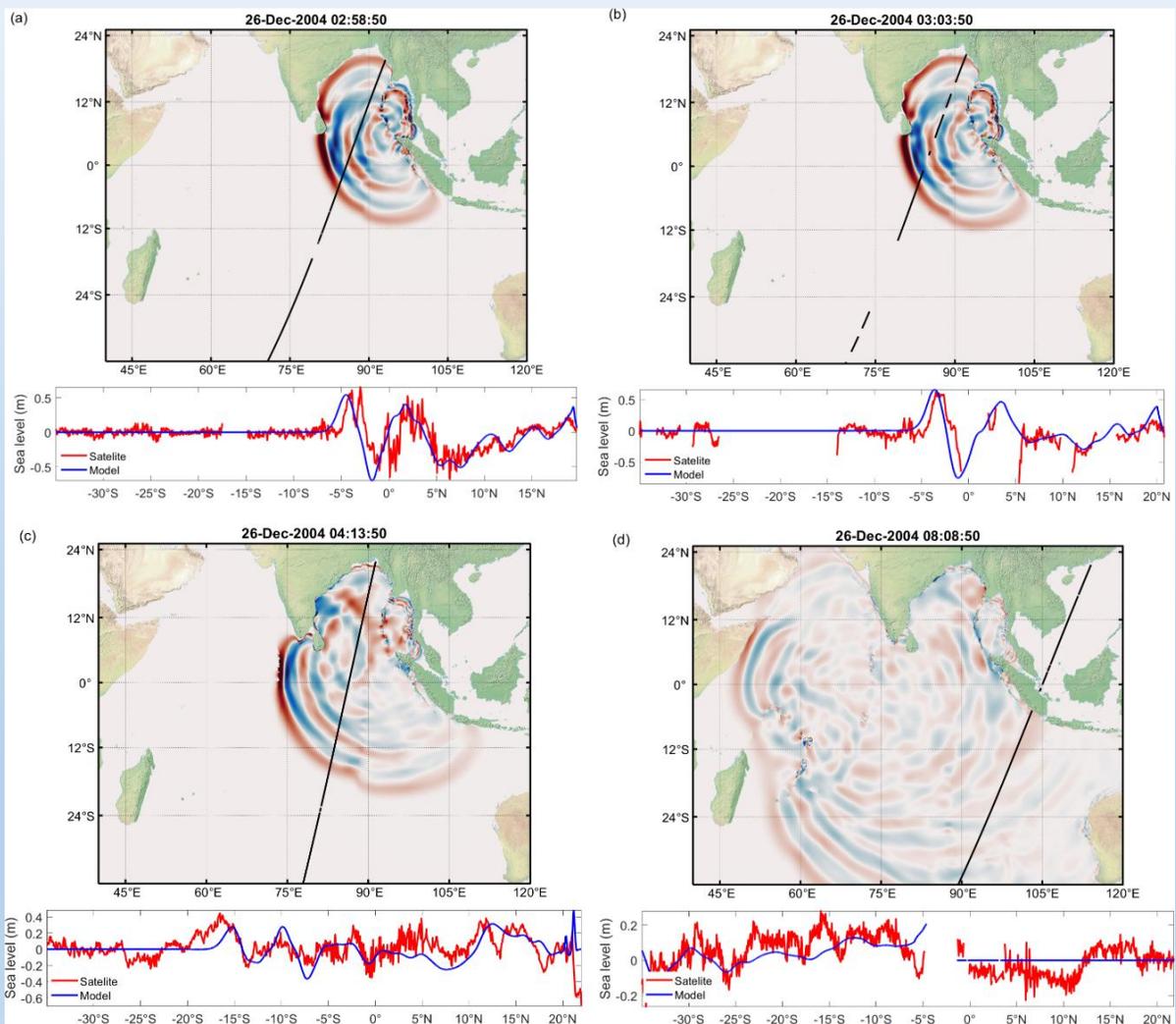
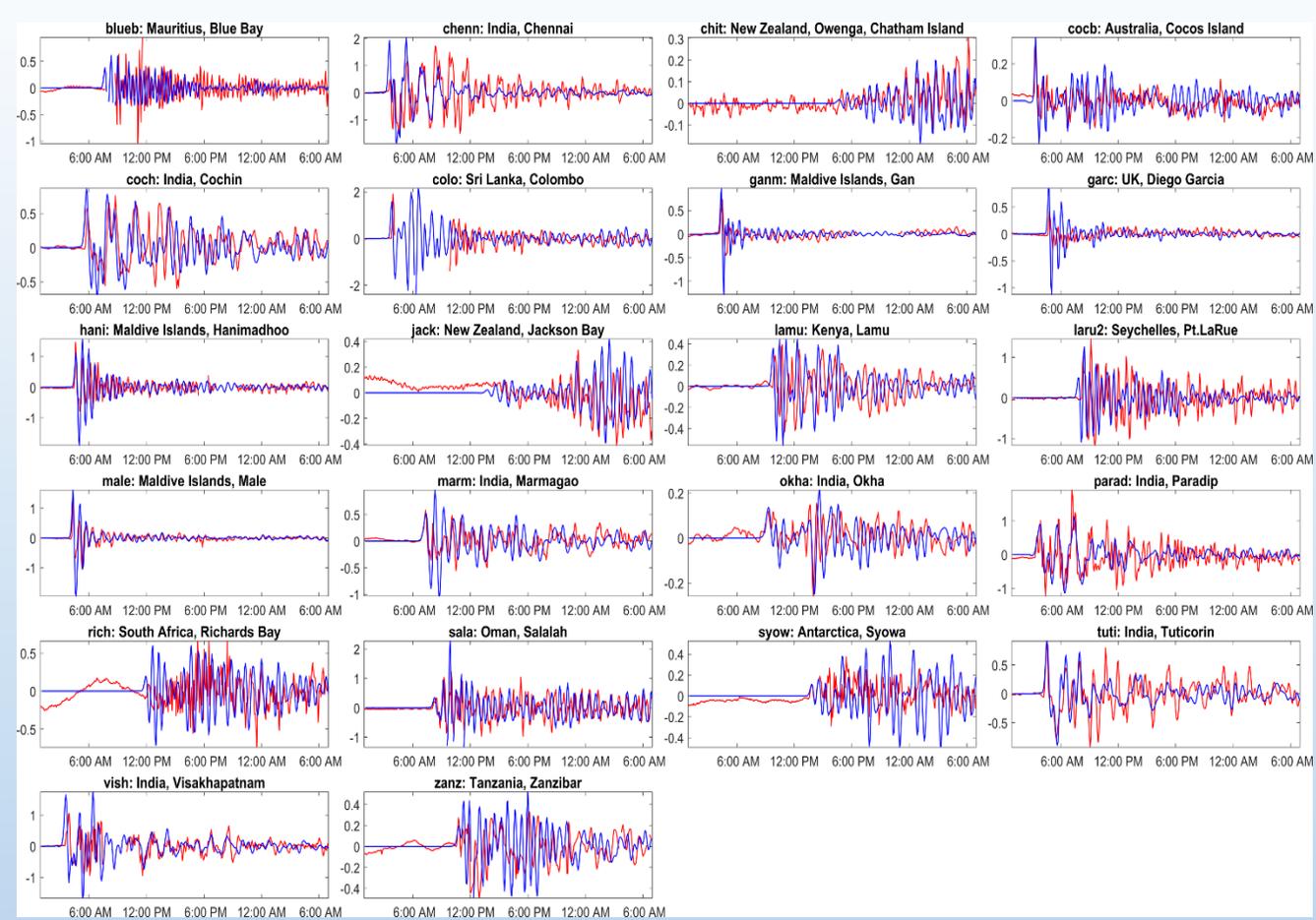
# Tsunami modelling over global oceans

Model domain and bathymetry



0 hours 2 mins





# Thank you